Calculus - Based Derivation of Least Squares Formulas p. 688 Modern Business Statistices 5e. (P.1)

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(1) Least squares method is a procedure for determining b_1 & b_0 that minimizes the sum of the squared residuals

(2) Sum of Square of Residual = $\sum (y_i - \hat{y}_i)^2$

(3) substitute $\hat{y}_i = b_0 + b_1 x_i$

↓

$$\sum (y_i - b_0 - b_1 x_i)^2$$

(4) take partial Derivatives w/ respect to b_1 & b_0

$$\frac{\partial \sum (y_i - b_0 - b_1 x_i)^2}{\partial b_0} = -2 \sum (y_i - b_0 - b_1 x_i)$$

$$\frac{\partial \sum (y_i - b_0 - b_1 x_i)^2}{\partial b_1} = -2 \sum x_i (y_i - b_0 - b_1 x_i)$$

(5) set two equations equal to zero (P.2)

$$-2\sum (y_i - b_0 - b_1 x_i) = 0 \qquad [1]$$

$$-2\sum x_i (y_i - b_0 - b_1 x_i) = 0 \qquad [2]$$

(6) Divide by 2 & summing each term individually:

[1]

$$-\sum y_i + \sum b_0 + \sum b_1 x_i = 0$$

(7) Bring $\left(-\sum y_i\right)$ to other side & noting that

$$\sum b_0 = n b_0$$

$$n b_0 + \left(\sum x_i\right) b_1 = \sum y_i \qquad [3]$$

(8) Similar Algebra to [2] yields:

$$\left(\sum x_i\right) b_0 + \left(\sum x_i^2\right) b_1 = \sum x_i y_i \qquad [4]$$

(9)

[3] & [4] are know as "Normal Equations"

(10)

$$N b_0 + \left(\sum x_i\right) b_1 = \sum y_i \rightarrow b_0 = \frac{\sum y_i - \left(\sum x_i\right) b_1}{n}$$

$$b_0 = \frac{\sum y_i}{n} - \frac{\left(\sum x_i\right)}{n} * b_1 \quad [5]$$

(11) substitute [5] into [4] yields:

$$\frac{\sum x_i \sum y_i}{n} - \frac{\left(\sum x_i\right)^2}{n} * b_1 + \left(\sum x_i^2\right) b_1 = \sum x_i y_i$$

(12) solving for b_1

$$b_1 = \frac{\sum x_i y_i - \left(\sum x_i \sum y_i\right)/n}{\sum x^2 - \left(\sum x_i\right)^2/n} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

(13) $b_0 = \bar{y} - b_1 \bar{x}$