Chapter 9: Hypothesis Testing

Hypothesis

- A statement about a population parameter subject to varification.
- Example:

  An official report claims:
  "The yearly salary of full-time realtor is $85,000."

Hypothesis Testing

1. A statistical procedure that uses sample evidence & probability theory to determine whether a statement about the value of a population parameter:

   "should be rejected" = "Reject"

   or

   "should NOT be rejected" "Fail to Reject"

   AND

2. Make a concluding statement about the population parameter based on sample evidence.

Example 1:

Statement from official Report:

"The yearly salary earned by full-time realtors is \$85,000"

Researcher believes:

Realtors make more than \$85,000

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1. If we take a random sample & get $\bar{x} = \$88,595$
2. we must decide if sampling Error of $88,595 - 85,000 = 3,595$ is acceptable.

Is the difference \$3,595

"Statistically significant"

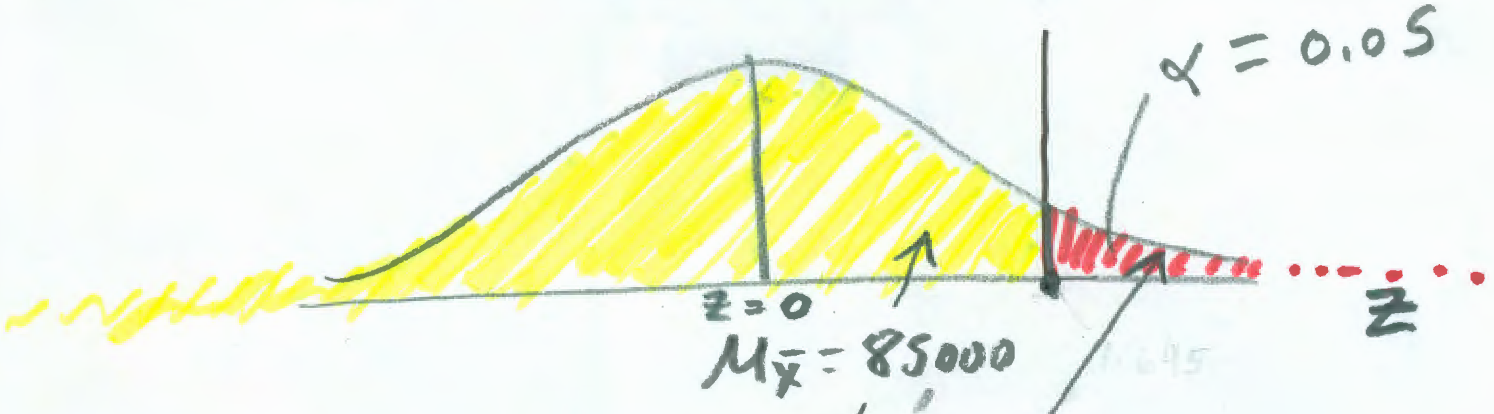
or

"Statistically insignificant"

chapter 7

Sampling Distribution of $\bar{x}$

Hurdle Line



If our sample evidence provided $\bar{x} = 86{,}000$

Then original claim of 85,000:

"should not be rejected"

"original claim seems reasonable"

If sample evidence provide $\bar{x} = 88{,}595$

Then original claim of 85,000:

"should be rejected"

"original claim seems unreasonable"

we will use something called a "Test statistic" (Z or t)

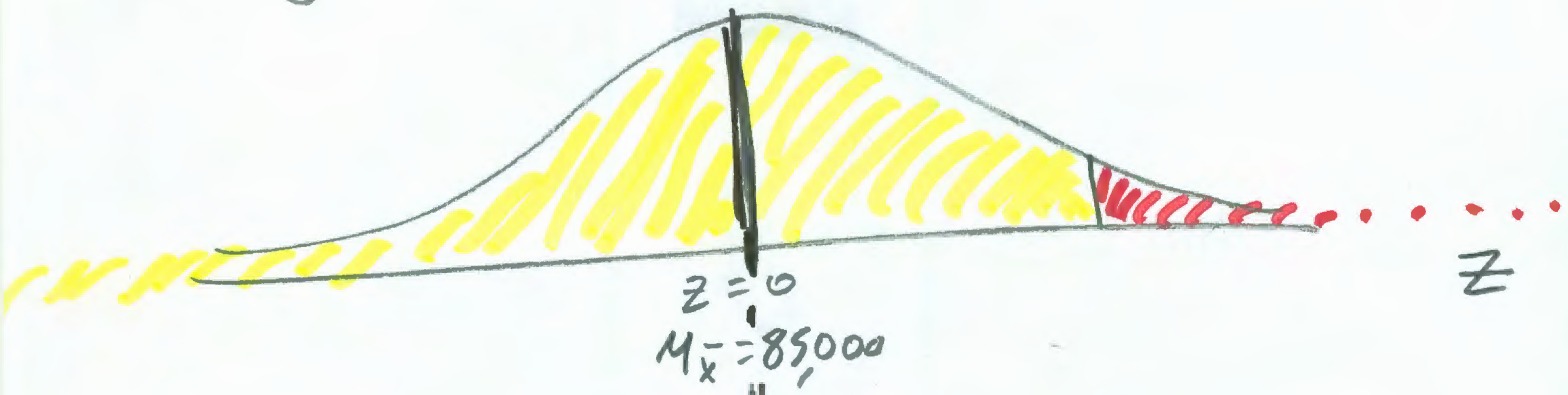
& compare it to our

Hurdle Line.

"Test statistic" = # standard Deviations above or below

Chapter 8

sampling Distribution of $\bar{x}$



if our sample evidence provided $\bar{x} = 86,000$

[$\bar{x}$ 86,000]

Then because interval contains 85,000, original claim:

"should Not be rejected"

or

"original claim seems reasonable"

if our sample evidence provided $\bar{x} = 88,595$

Then because interval does not contain 85,000,

[$\bar{x}$ 88,595]

original claim:

"should be rejected"

or

"original claim seems unreasonable"

other examples of "statements about a value of a population parameter" that we can test: 5

Is the new contribution solicitation letter more effective than the old letter, which got 15% contributions?

Is the manufacturer's claim that 16 oz. of catsup is in each bottle?

Is the average wait time in line at McBurger's Resturant less than 3 minutes?

Is the new machine faster than the old one?

Steps of Hypothesis Testing

① Develop Null Hypothesis (H_0) & Alternative Hypothesis (H_1) or (H_a)

② specify the level of significance (α)

③ collect sample Data & compute value of test statistic (Z or t), Draw Picture.

P-value Approach

④ use value of test statistic to compute p-value

⑤ Reject H_0 if p-value $\leq \alpha$

Critical Value Approach

④ use level of significance to determine the critical value and state rejection rule

⑤ use the value of the test statistic and the rejection rule to determine whether to reject H_0

Notes: ① If population data is normally distributed, these methods are exact (.99 = CI, α=.01, then 99 intervals contain mu, 1 does not)

② If pop. data is not Normal, the bigger the n, the more exact.

Pop Normal	= any n can be used
Appox. Normal	$n \geq 15$
Not Normal	$n \geq 30$
outliers	$n \geq 50$

Step 1

Develop Null Hypothesis (H_o) & Alternative H_o (H_a)

Null Hypothesis = H_o

The hypothesis tentatively assumed true in the hypothesis testing procedure. Based on sample evidence we either

"Reject H_o"

or

"Fail to Reject H_o"

Alternative Hypothesis = H_a

Based on sample evidence the hypothesis concluded to be true if the null hypothesis is rejected.

we either

"Fail to Reject H_o"

or

"Reject H_o, accept H_a"

Research Hypothesis

Start with alternative hypothesis and make it the conclusion the researcher hopes to support.

Example:

Realtors make more than \$85,000

Validity of a Claim

Assumption that population Parameter is true

Example:

Is catsup bottle filled with 16 oz.?

Decision Making

Choose between 2 things.

Example:

Should we accept box of shipped products, yes or No.

Notes about Step I

- Developing H_0 & H_a can be difficult & takes practice to learn how to do.

- The context, situation, or point of view will help determine the correct H_0 & H_a

1. **Research Hypothesis** → usually start with → H_a

   Example: "Realtors make more than \$85,000?"

   $H_0: \mu \leq 85{,}000$

   $H_a: \mu > 85{,}000$

2. **Validity of Claim** → usually start with → H_0

   Example: "Is catsup bottle filled with 16oz."?

   Consumers' point of view

   Assume bottle is filled with 16oz. But if not, take action

   $H_0: \mu \geq 16$ oz.

   $H_a: \mu < 16$ oz.

   Consumer cares about this

   or

   Manufacturers' point of view

   Manufacture cares about filled too much or too little

   $H_0: \mu = 16$ oz.

   $H_a: \mu \neq 16$ oz.

3. **Decision Making** → (choose between 2 things) → H_0 or H_a

step 1 Develop H_0 & H_a

original statement:

The yearly salary earned by full-time realtor is \$85,000 (sigma = \$12,549)

competing statement:

Researcher believes realtors make more than \$85,000

1st write this:

colon says "Here is Hypothesis"

$H_0 : \mu$

$H_a : \mu$

2nd : Use "more than \$85,000" to determine comparative operator for H_a

$H_0 : \mu$

$H_a : \mu > 85{,}000$

3rd : once you know comparative operator for H_a, put opposite comparative operator <u>and</u> equal sign for H_0.

4th H_0 ALWAYS get = sign

5th H_a comparative operator

$H_0 : \leq 85{,}000$

$H_a : > 85{,}000$

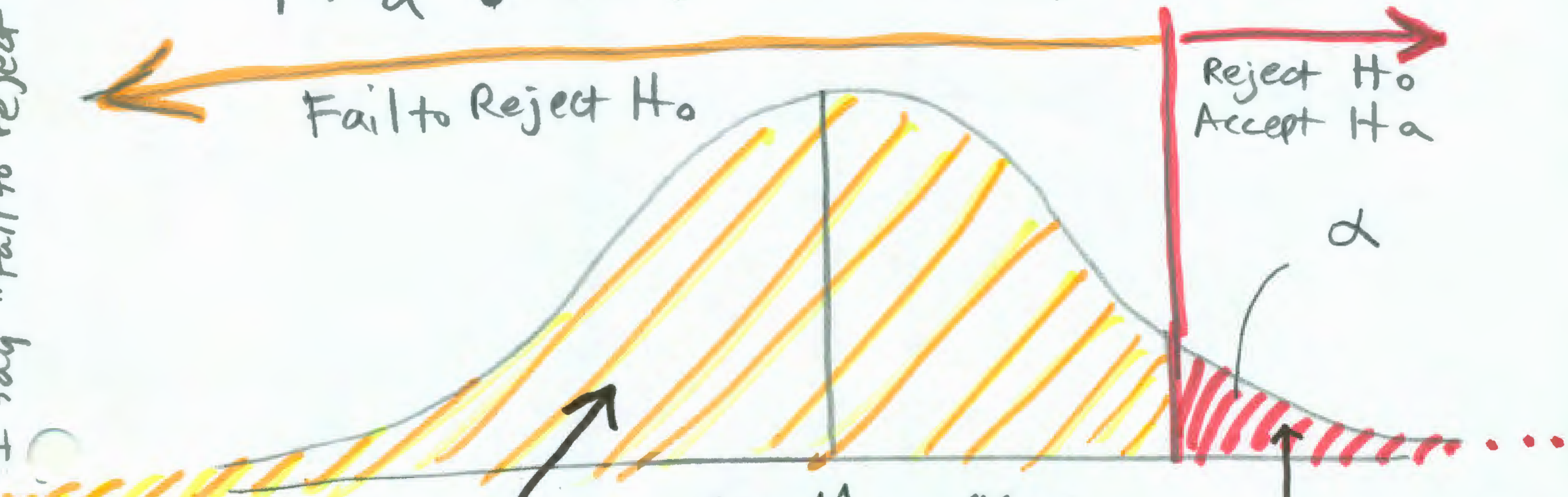
1-tail to Left.

Always tells you which way test is

Step 1 continued..

$$H_o: \mu \leq \$85,000$$

$$H_a: \mu > \$85,000$$



$\mu = \mu_{\bar{x}} = 85,000$

$$\text{Standard Error} = \frac{12,549}{\sqrt{n}}$$

If we get $\bar{X}$ here we say:

"Based on the sample evidence, we fail to reject Ho. There is little statistical evidence that the mean salary is more than \$85,000." *Don't say Ho is TRUE

If we get $\bar{X}$ here we say:

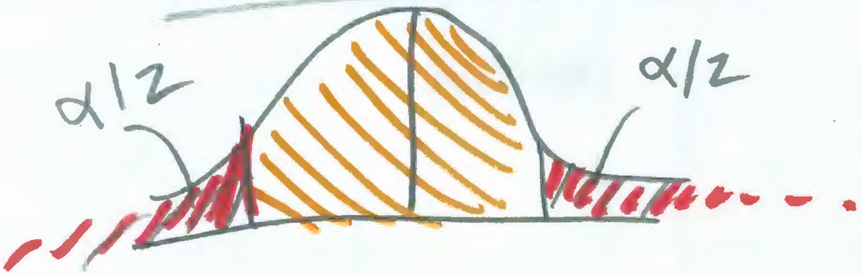
"Based on the sample evidence, we reject Ho and accept Ha. There is statistical evidence that the mean salary is more than \$85,000."

- careful in our language because we are taking samples.
- only two possible outcomes.

Note* Book says "Do not reject Ho". I say "Fail to reject Ho".

3 possible Forms of H_0 & H_a

1



2-tail

$$H_0 : \mu = \mu_0$$

$$H_a : \mu \neq \mu_0$$

2

1-tail to Right

$$H_0 : \mu \leq \mu_0$$

$$H_a : \mu > \mu_0$$

3

1-tail to Left

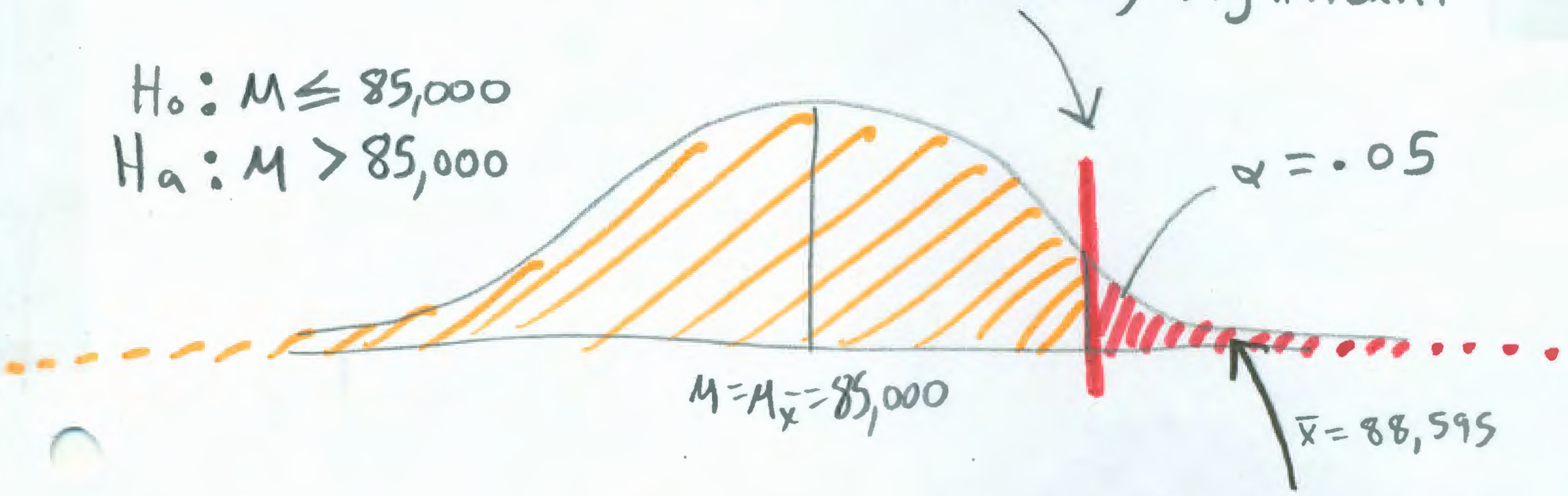
$$H_0 : \mu \geq \mu_0$$

$$H_a : \mu < \mu_0$$

$\boxed{\text{Level of significance} = \text{Alpha} = \alpha}$ ← procedural definition

1. α determines the cut off point, which is the threshold used to decide whether the test statistic is statistically significant

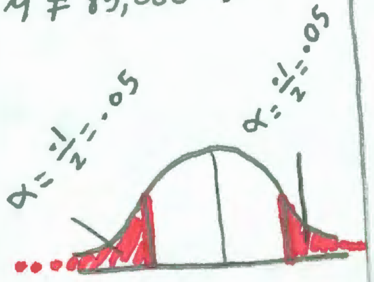
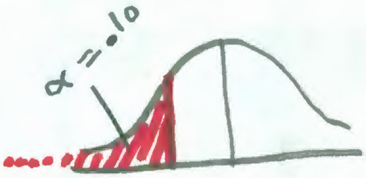
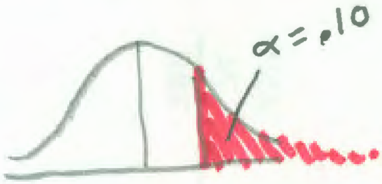
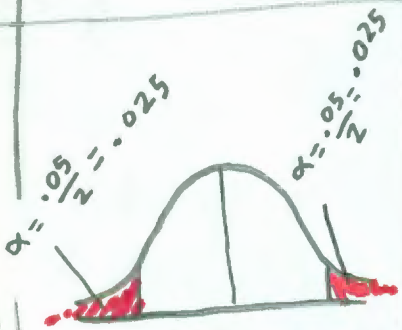
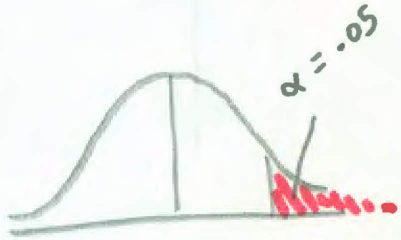
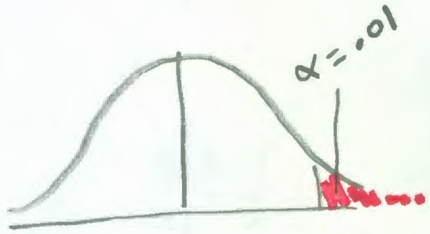
$H_0: \mu \leq 85{,}000$
$H_a: \mu > 85{,}000$



If we get $\bar{x} = 88{,}595$ and it is out here, this is statistically significant and we reject H_0 and accept H_a. $\mu > 85{,}000$.

- If we choose $\alpha = 0.05$, we are taking a 5% risk of rejecting H_0 even though it was true.
- Because we choose α, we can say we are doing a "Significance Test"

picture Examples for Level of significance where $\mu = 85,000$

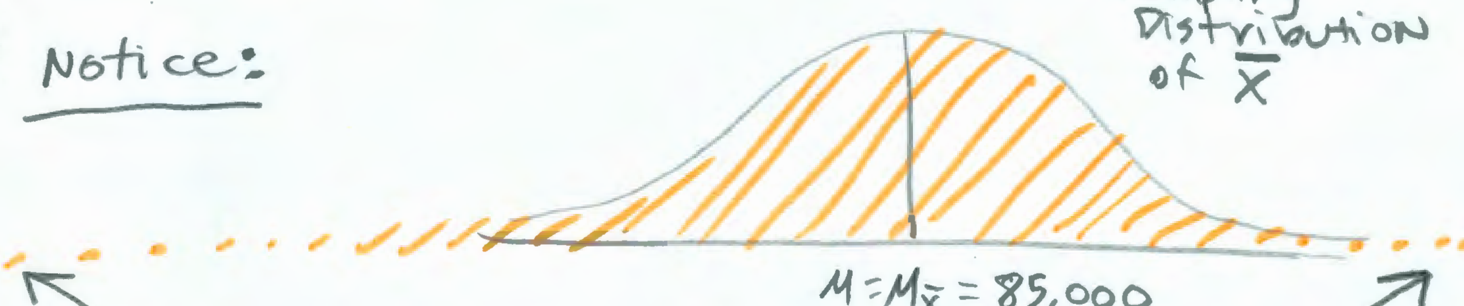
Alpha	2-tail If Question was "is $\mu \neq 85,000$".	1-tail to Left If Question was "is $\mu < 85,000$".	1-tail to Right If Question was "is $\mu > 85,000$".
$\alpha = .10$	 $H_0: \mu = 85000$ $H_a: \mu \neq 85000$	 $H_0: \mu \geq 85000$ $H_a: \mu < 85000$	 $H_0: \mu \leq 85000$ $H_a: \mu > 85000$
$\alpha = .05$	 $H_0: \mu = 85000$ $H_a: \mu \neq 85000$	 $H_0: \mu \geq 85000$ $H_a: \mu < 85000$	 $H_0: \mu \leq 85000$ $H_a: \mu > 85000$
$\alpha = .01$	 $H_0: \mu = 85000$ $H_a: \mu \neq 85000$	 $H_0: \mu \geq 85000$ $H_a: \mu < 85000$	 $H_0: \mu \leq 85000$ $H_a: \mu > 85000$

These picture examples show the 3 possibilities at 3 different alpha values.

Step 2 Specify Level of Significance (α)

- Because hypothesis Testing is based on Sample Data, we must allow for the possibility of errors.
- unless we test whole population, you run risk of error.

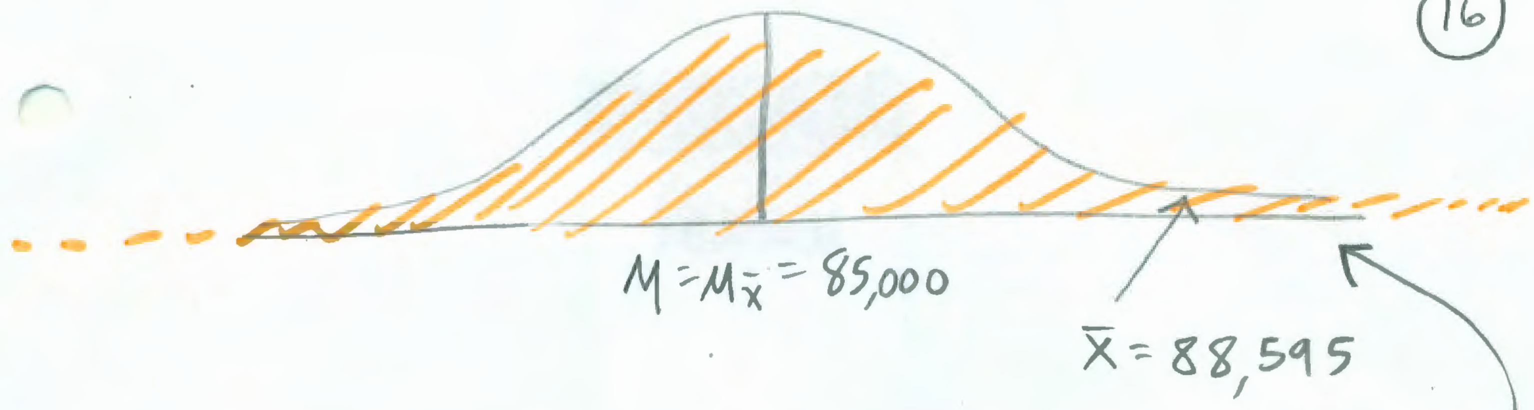
① Notice:



This is entire distribution of possible $\bar{X}$ values.

② It is possible to take a sample & get an $\bar{X} = 88,595$ that is just sample error:

So we could get $\bar{X} = 88,595$
But $\mu = 85,000$ is still true



It is very unlikely that we could get an $\bar{x}$ out here.

But...

It is a possibility that our particular sample just happened to have a lot of big numbers in it so we got:

$$\bar{x} = \$88{,}595$$

while the full population mean was still:

$$\mu = \$85{,}000$$

So...

Because we can't usually test the whole population, we have to pick a cut off point and reject the original statement (H_0) if we go beyond that point.

→ This means we take a risk of an error...

Define:

↙ definition (what it is)

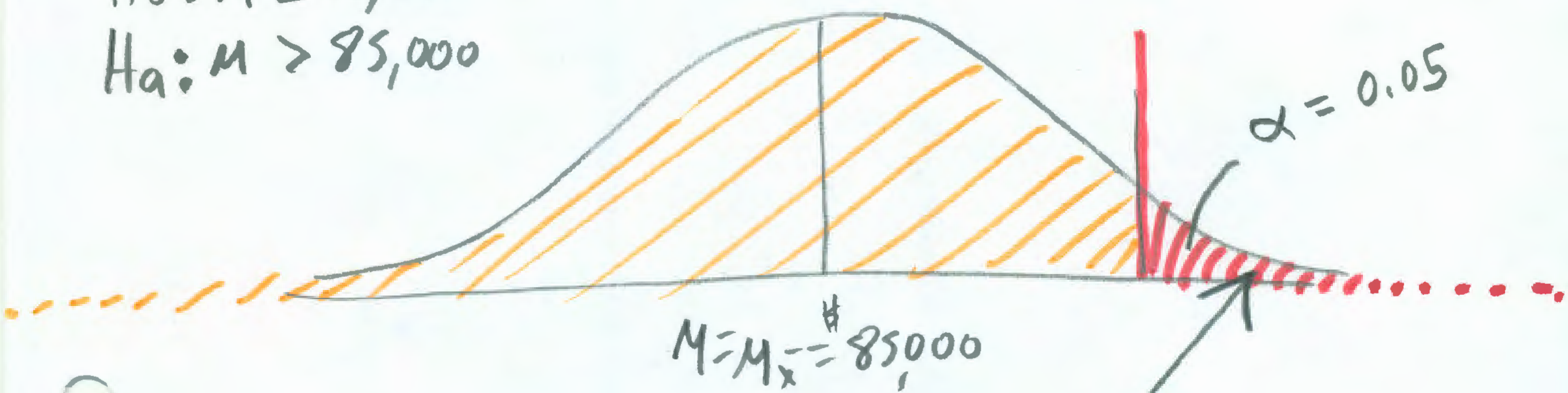
$\boxed{\text{Level of Significance} = \text{Alpha} = \alpha = \text{Type I Error}}$

Probability (Risk) of rejecting H_0 even though it is True (as an equality).

example: $\mu = 85{,}000$

$H_0: \mu \le 85{,}000$

$H_a: \mu > 85{,}000$



If we get $\bar{x} = \$88{,}595$ & Reject H_0,

But H_0 ($\mu = 85{,}000$) is actually TRUE this is:

"Type I Error"

"Innocent but found Guilty"

(★) Most of the time $\bar{x} = 88{,}595$ will lead to correct conclusion (about 95 out of 100).

(★) "Innocent but found Guilty" Error 5 out of 100 times.

Designer of Hypothesis Test selects alpha and thereby controls the probability of a Type I Error:

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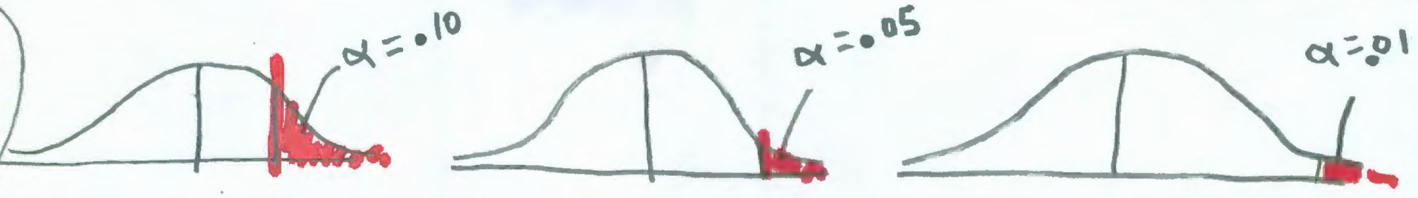
especially as n gets big



About: Error 10 out of 100 times | Error 5 out of 100 times | Error 1 out of 100 times

→ As you Move ⟶ this way → you reduce α →
→ and Reduce ⟶ the risk of Type I Error →

Example: ① Drug company may want to set α very small, so they are sure New Drug really works

② Quality Control may want to set α low so they are more sure that the quality is high.

Selecting α

- If cost of Making Type I Error is high, choose small α
- If cost of Making Type I Error is Not high, choose bigger α

Errors & Correct Conclusions in Hypothesis Testing

		Actual Population Condition	
		H_0 TRUE (H_a FALSE)	H_0 FALSE (H_a TRUE)
Conclusion (based on Sample)	Reject H_0, Accept H_a	Type I Error Alpha	Correct Conclusion
	Fail To Reject H_0	Correct Conclusion	Type II Error Beta

Type I Error

H_0 True, But we Reject H_0

Alpha $= \alpha$

"Innocent but found Guilty"

Type II Error

H_0 False, But we Fail to Reject H_0

Beta $= \beta$

"Guilty but found innocent"

Because we control for α we can say "Accept H_a" in our conclusion

Because we don't control for β (In this textbook) we can't say "Accept H_0"

Other Wording:

		Actual Population Condition	
		H_0 TRUE (H_a FALSE)	**H_0 FALSE (H_a TRUE)**
Conclusion (based on Sample)	**Reject H_0 ,Accept H_a**	Type I Error Alpha (Level of Significance) **"False Positive"**	Correct Conclusion **"True Positive"**
	Fail To Reject H_0	Correct Conclusion **"True Negative"**	Type II Error Beta **"False Negative"**

False Positive: Our Alternative (H_a) was selected even though the Null (H_0) was true.

False Negative: Our Alternative (H_a) was not selelcted even though the Null (H_0) was false.

Step 3 | Collect sample Data, calculate value of Test statistic (z or t)

Example 1:

Step 1 $H_0: \mu \leq 85{,}000$ Annual Realtor Salary

$H_a: \mu > 85{,}000$ Annual Realtor Salary

Step 2 α = Type I Error = 0.05 = (Cost of error Not too big)

Step 3 we go out & get a sample

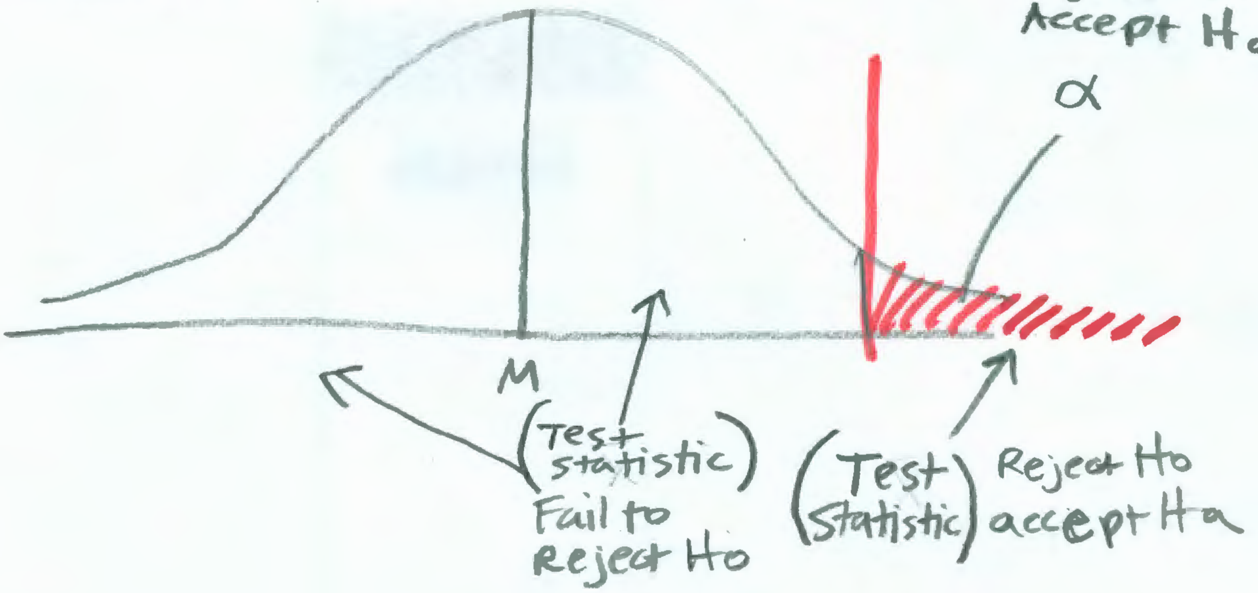
$\bar{X} = 88{,}595$

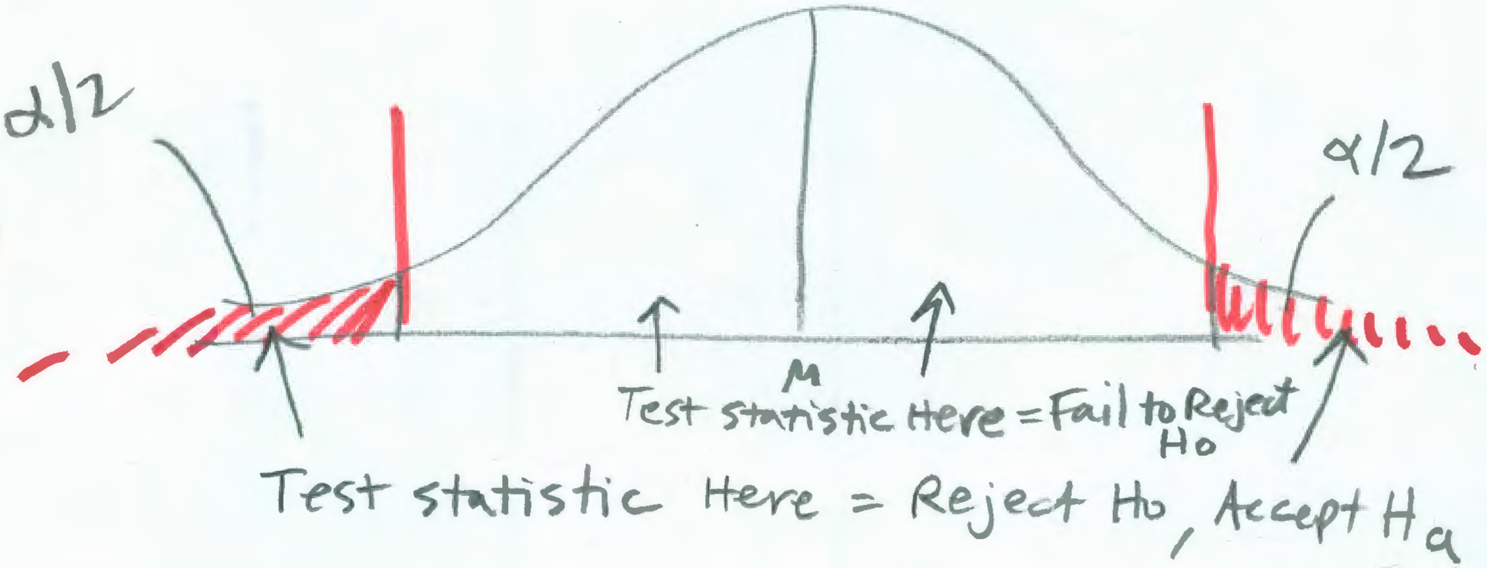
sigma known = 12,549

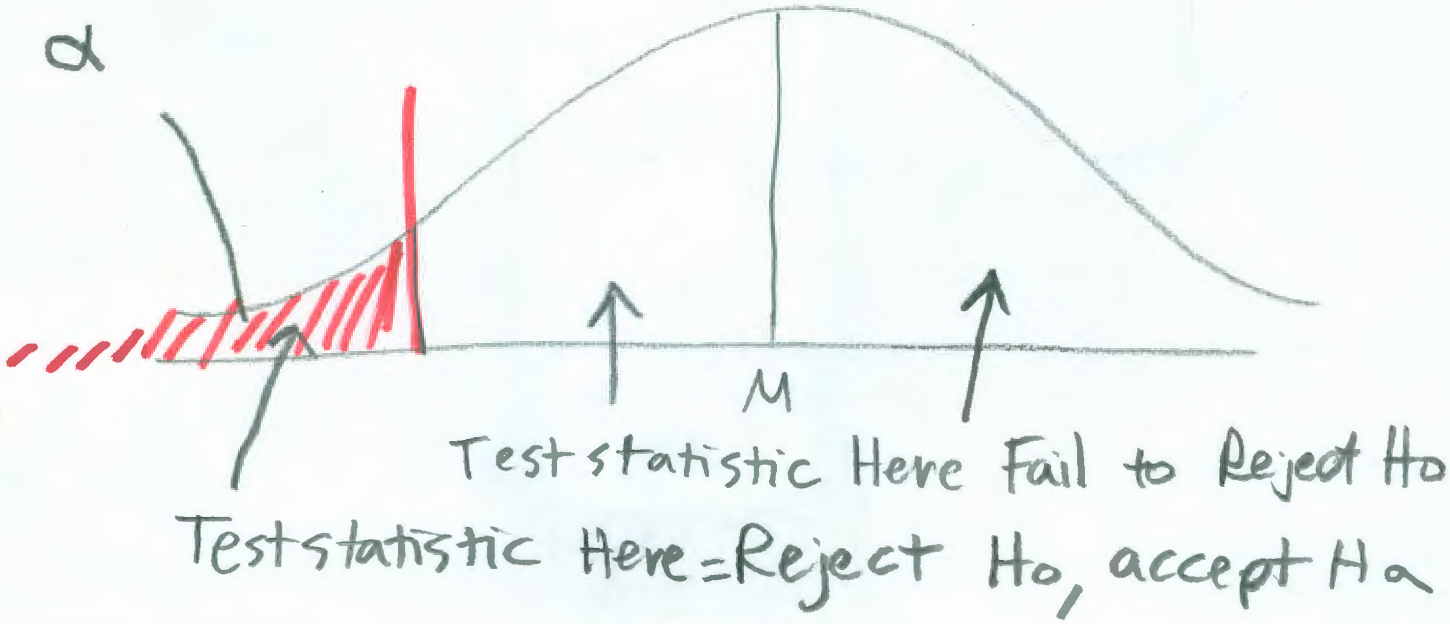
$n = 36$ (Big enough to accomidate some outlier salaries)

But we Need test statistic

1. α determines cut off point
2. test statistic beyond cut off point, then we reject Ho Accept Ha





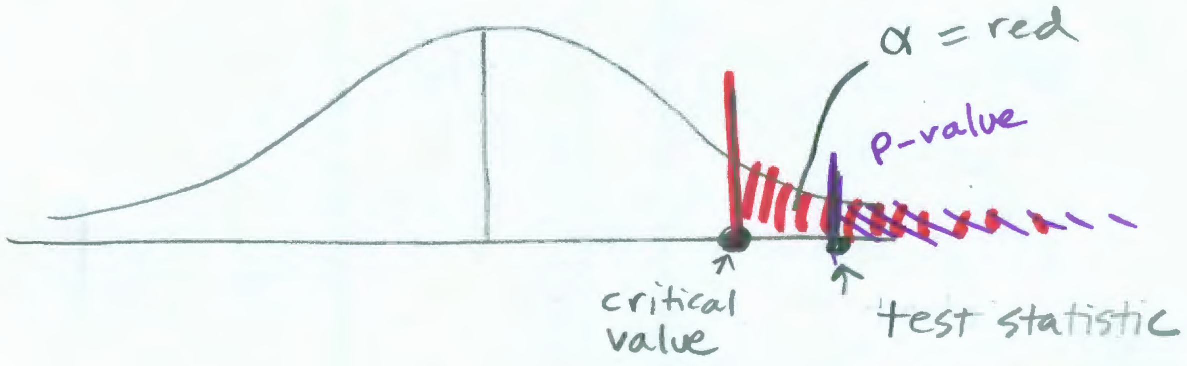


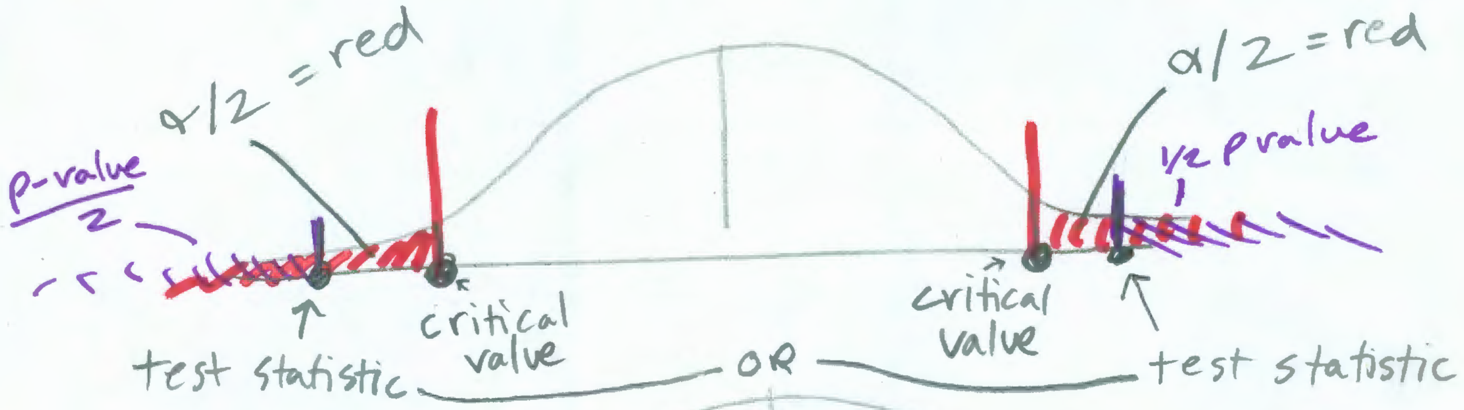
2 Methods for determining whether test statistic is past cut off ("Statistically significant")

① p-value : p-value ≤ α Reject H_o, Accept H_a

or

② critical value : If test statistic is past critical value Reject H_o, Accept H_a





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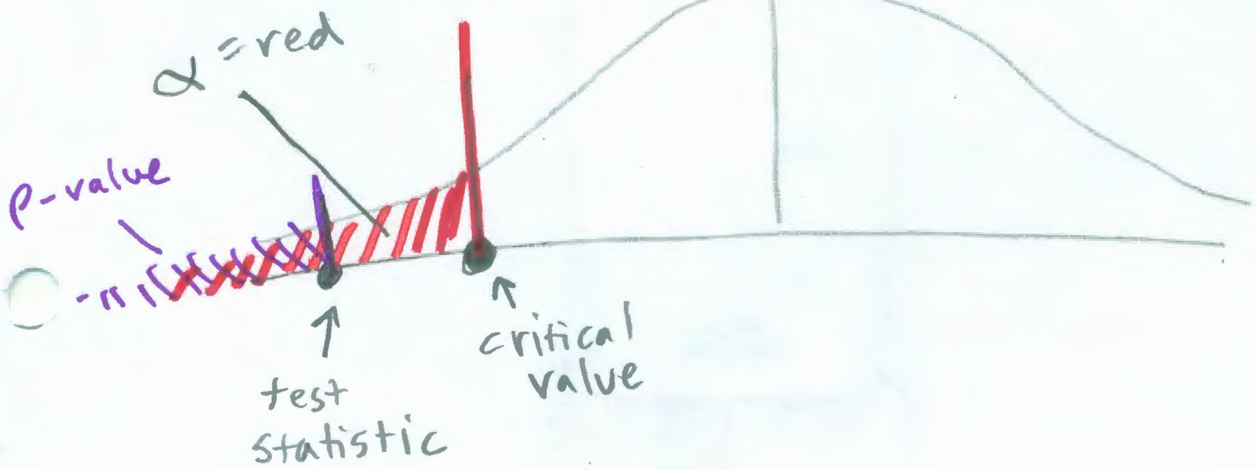
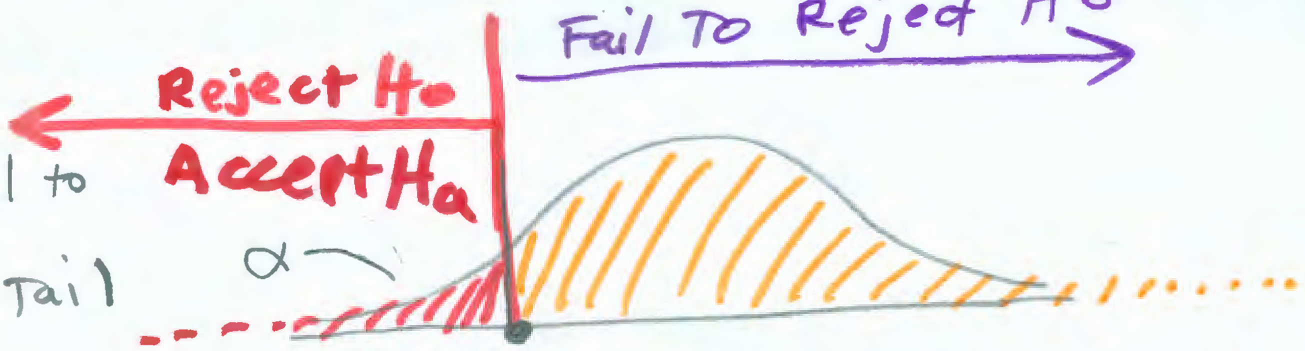


critical value

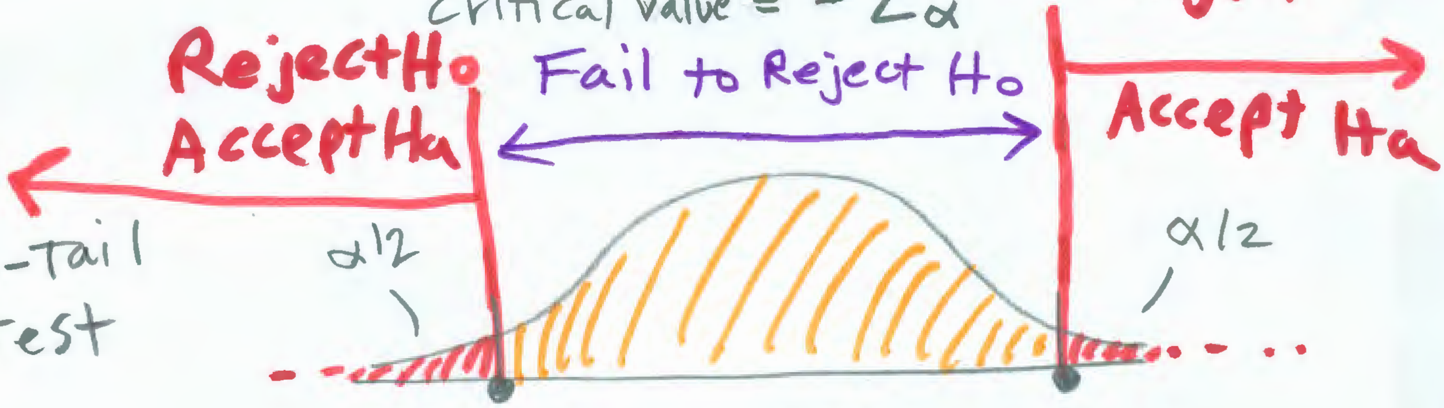
Hurdle point that determines if the Null Hypothesis is Rejected & the Alternative Hypothesis is Accepted. calculate critical value based on Alpha

1-tail to Left Lower Tail



critical value = $-Z_\alpha$

2-Tail Test



$-Z_{\alpha/2}$ — critical values — $Z_{\alpha/2}$

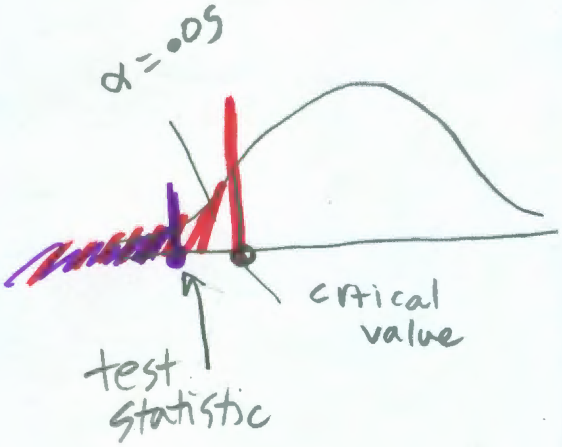
1-tail to Right upper Tail

Z_α = critical value

p-value "observed level of significance"

Probability of getting the test statistic value or worse (less or more).

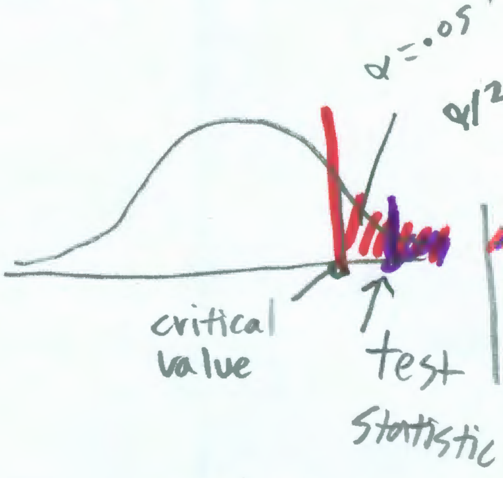
1 tail to Left



p-value = probability of getting test statistic or less

p-value * 1

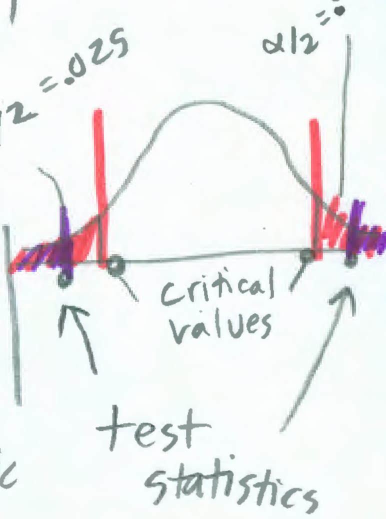
1 tail to Right



p-value = probability of getting test statistic or more

P-value * 1

Two tail



P-value from 1 side = probability of getting Test statistic or worse (Less or more)

then,

P-value * 2

Rejection Rule: p-value ≤ α, Reject Ho, Accept Ha

Interpreting p-value

p-value > 0.10	Insufficient evidence to say Ha True
0.05 < p-value ≤ 0.10	weak evidence to say Ha True
0.01 < p-value ≤ 0.05	strong evidence to say Ha True
p-value ≤ 0.01	overwhelming evidence to say Ha True

Advantage of p-value (over critical value) is that it tells you how significant the results are:

1. what probability of getting a test statistic or worse (less or more)
2. what the Type I Error Rate is, like we got \$88,595 for $\bar{x}$ & that value would appear as a True sample mean w/ $\mu = 85{,}000$ 4 in 100 times.

Test statistic (z or t) for Hypothesis Testing About a population mean

σ KNOWN	σ NOT KNOWN
$z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

μ_0 = hypothesized mean

z & t = calculated test statistic, used to determine whether to reject the Null Hypothesis. Compare z or t to critical value to make decision, or used to calculate p-value. z & t = number of Standard Errors above/below Hypothesized mean.

$\bar{x}$ = sample mean

σ = population standard deviation

s = sample standard deviation

n = sample size

Test statistic for Hypothesis Tests About A Population Proportion

$$Z = \frac{\bar{P} - P_0}{\sqrt{\frac{P_0 * (1-P_0)}{n}}}$$

$\bar{P}$ = sample proportion $= \frac{\text{successes}}{n}$

P_0 hypothesized pop. proportion

n = sample size

$$SE = \sigma_{\bar{P}} = \sqrt{\frac{P_0 * (1-P_0)}{n}}$$

Must verify:

1. Are there fixed # Trials?
2. Are results Independent?
3. Does each Trial result in success or Failure?
4. P stay same on each trial?
5. $n*p > 5$, $n*(1-p) > 5$ } textbook assumes true for all problems.

* since exact sampling Distribution of $\bar{P}$ (Pbar) is Descrete, small samples require additional steps that we will not do in this textbook.

Q: When are we allowed to use t Distribution?

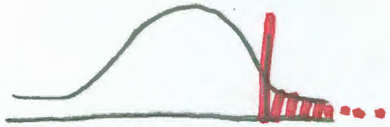
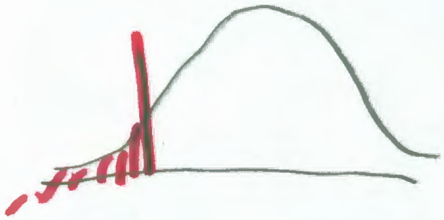
A: When population distribution in normally distributed or near normal, or n is sufficicently large enough

1) If pop distribution is normal or near normal, smaller than 30 sample size may be used
2) If pop distribution is not normal, n >= 30 usually adequate
3) If pop distribution is highly skewed or has outliers, n >= 50 should be used

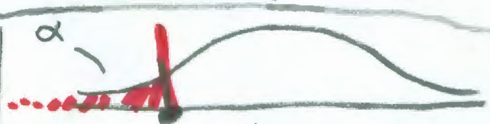
Notes:

If the population distribution is not known a histogram based on a sample may give you a clue.
Although histogram is not conclusive, sometimes it may be the best clue that you have.
The the histogram shows non-normal or outliers, increasing the sample size and improve the calculations.

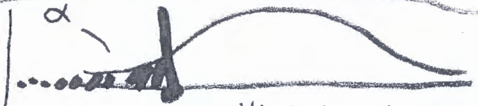
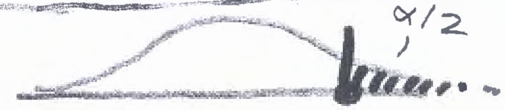
Excel Functions

	Z Distribution	t Distribution
1 tail to Right upper 	P-value = 1-NORM.S.DIST(Z,1) upper critical value = NORM.S.INV(1-α)	P-value = 1 - T.DIST(t, df, 1) or = T.DIST.RT(t, df) upper critical value = T.INV(1-α, df)
Two tail 	P-value = NORM.S.DIST(Z,1)*2 Z on Low end Low critical value = NORM.S.INV(α/2) upper = NORM.S.INV(1-α/2) or +/- critical values = +/-NORM.S.INV(α/2)	P-value = T.DIST(lower t, df, 1)*2 or = T.DIST.2T(upper t, df) Low critical value T.INV(α/2, df) upper = T.INV(1-α/2, df) or +/- critical values = +/- T.INV(α/2, df)
1 tail to Left upper 	P-value = NORM.S.DIST(Z,1) Low critical value = NORM.S.INV(α)	P-value = T.DIST(t, df, 1) Lower critical value = T.INV(α, df)
When to use:	sigma known and proportions, when 4 tests met.	sigma not known

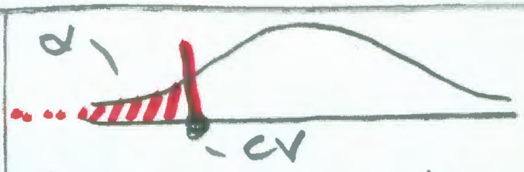
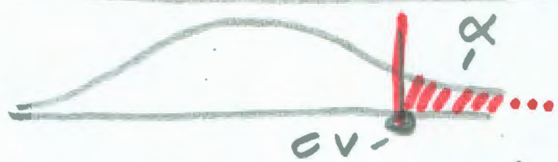
(Z) Hypothesis Testing Z Distribution (Sigma Known)

Test Type	 1 Tail Test To Left ← Lower Tail	 ← Two-tail Test →	 1 Tail Test to Right Upper Tail →
Hypothesis	$H_0: \mu >= \mu_0$ $H_a: \mu < \mu_0$	$H_0: \mu = \mu_0$ $H_a: \mu <> \mu_0$ * <> → Not Equal To	$H_0: \mu <= \mu_0$ $H_a: \mu > \mu_0$
Test statistic		$Z = \dfrac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$	
p-value Rejection Rule		If: p-value $\leq \alpha$ Then: Reject Ho, Accept Ha	
Excel p-value	=NORM.S.DIST(z,1)	=NORM.S.DIST(z,1)*2 z on Low End	=1-NORM.S.DIST(z,1)
critical value Rejection Rule (for 1 tail) Fail To Reject (for 2-Tail)	If: $Z \leq -Z_\alpha$ Then: Reject Ho, Accept Ha $-Z_\alpha$ = critical value (low End)	If: $-Z_{\alpha/2} < Z < Z_{\alpha/2}$ Then: Fail To Reject Ho $-Z_{\alpha/2}$ = Low critical value $Z_{\alpha/2}$ = Upper critical value	If: $Z \geq Z_\alpha$ Then: Reject Ho, Accept Ha
Excel critical value	$-Z_\alpha$ = NORM.S.INV(α)	+/− critical value = = NORM.S.INV(α/2)	Z_α = NORM.S.INV(1−α)

Ⓩ Hypothesis Testing Z Distribution (Proportions)

Test Type	 1 Tail Test To Left ← Lower Tail	 ← Two-tail Test →	 1 Tail Test to Right Upper Tail →
Hypothesis	Ho : $p >= p_0$ Ha : $p < p_0$	Ho : $p = p_0$ Ha : $p <> p_0$	Ho : $p <= p_0$ Ha : $p > p_0$
Test statistic	$\left\{\begin{matrix}\text{Standard}\\ \text{Error}\end{matrix}\right\} = SE = \sigma_{\bar{p}} = \sqrt{\frac{p_0 * (1-p_0)}{n}}$ / $Z = \frac{\bar{p} - p_0}{SE}$		
p-value Rejection Rule	If: p-value $\leq \alpha$ Then: Reject Ho, Accept Ha		
Excel p-value	=NORM.S.DIST(z, 1)	=NORM.S.DIST(z, 1)*2 z on Low End ↗	=1 -NORM.S.DIST(z,1)
critical value Rejection Rule (For 1 tail) Fail To Reject (for 2-Tail)	If: $Z \leq -Z_\alpha$ Then: Reject Ho, Accept Ha $-Z_\alpha$ = critical value (low End)	If: $-Z_{\alpha/2} < Z < Z_{\alpha/2}$ Then: Fail TO Reject Ho $-Z_{\alpha/2}$ = Low critical value $Z_{\alpha/2}$ = Upper critical value	If: $Z \geq Z_\alpha$ Then: Reject Ho, Accept Ha
Excel critical value	$-Z_\alpha$ =NORM.S.INV(α)	+/− critical value = =NORM.S.INV (α/2)	Z_α =NORM.S.INV(1−α)

(t) Hypothesis Testing t Distribution (Sigma Not Known)

Test Type	 ← Lower Tail 1 Tail Test To Left	 ← Two-tail Test →	 Upper Tail → 1 Tail Test o Right
Hypothesis	$H_0: \mu >= \mu_0$ $H_a: \mu < \mu_0$	$H_0: \mu = \mu_0$ $H_a: \mu <> \mu_0$	$H_0: \mu >= \mu_0$ $H_a: \mu > \mu_0$
Test statistic	$t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$		
P-value Rejection Rule	If: P-value $\leq \alpha$ Then: Reject H_0, Accept H_a		
Excel P-value	=T.DIST(t,df,1)	=T.DIST.2T(upper t,df) or =T.DIST(lower t,df,1)*2	=1-T.DIST(t,df,1) or =T.DIST.RT(t,df)
Critical value Rejection Rule (for 1 Tail) Accept Rule (for 2-tail)	If: $t \leq -t_\alpha$ Then: Reject H_0, Accept H_a $-t_\alpha$ = Low Critical Value	If: $-t_{\alpha/2} < t < t_{\alpha/2}$ Then: Fail to Reject H_0 $-t_{\alpha/2}$ = Low critical value $t_{\alpha/2}$ = Upper Critical value	If: $t \geq t_\alpha$ Then: Reject H_0, Accept H_a t_α = upper Critical Value
Excel Critical Value	$-t_\alpha$ =T.INV(α,df)	Lower =T.INV($\alpha/2$,df) upper =T.INV($1-\alpha/2$,df) or +/- =T.INV(α,df)	t_α= T.INV($1-\alpha$,df)

Example of step 3 (collect Data, calculate test statistic, Draw Picture)

Because we know the population standard deviation $\sigma = 12{,}549$, we can use the test statistic, Z.

$n = 36$

$\bar{x} = \$88{,}595$

$\sigma = \$12{,}549$

$\mu = \mu_{\bar{x}} = 85{,}000$

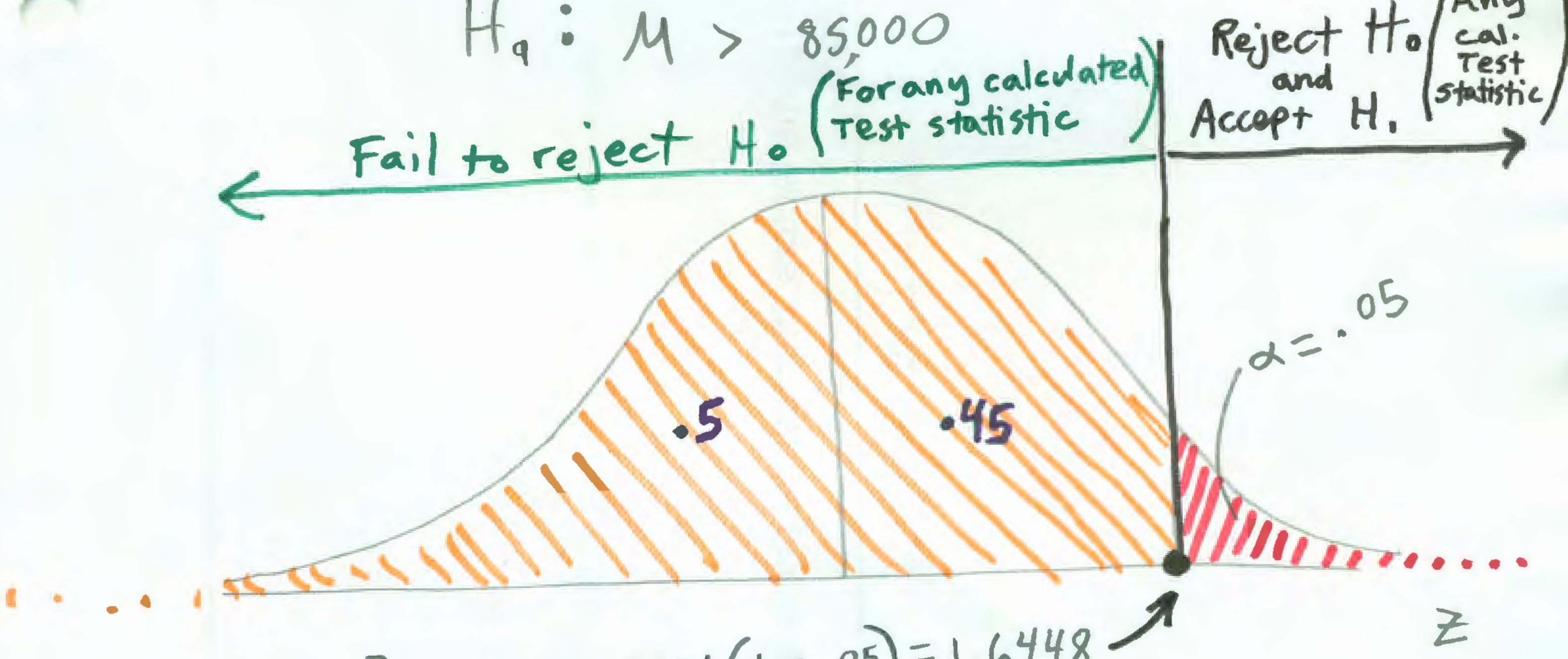
$\sigma_{\bar{x}} = \frac{12549}{\sqrt{36}} = \$2{,}091.50$

$\alpha = .05$

test statistic $= \frac{88595 - 85000}{2{,}091.50} = 1.72$

$H_0 : \mu \leq 85{,}000$

$H_a : \mu > 85{,}000$



$\left\{\begin{matrix}\text{critical} \\ \text{value}\end{matrix}\right\} = \text{NORMS.INV}(1 - .05) = 1.6448$

$\bar{x}$

$\mu = \mu_{\bar{x}} = 85000$

$\sigma_{\bar{x}} = 2{,}091.50$

Step 4 — **Decision Rule**:

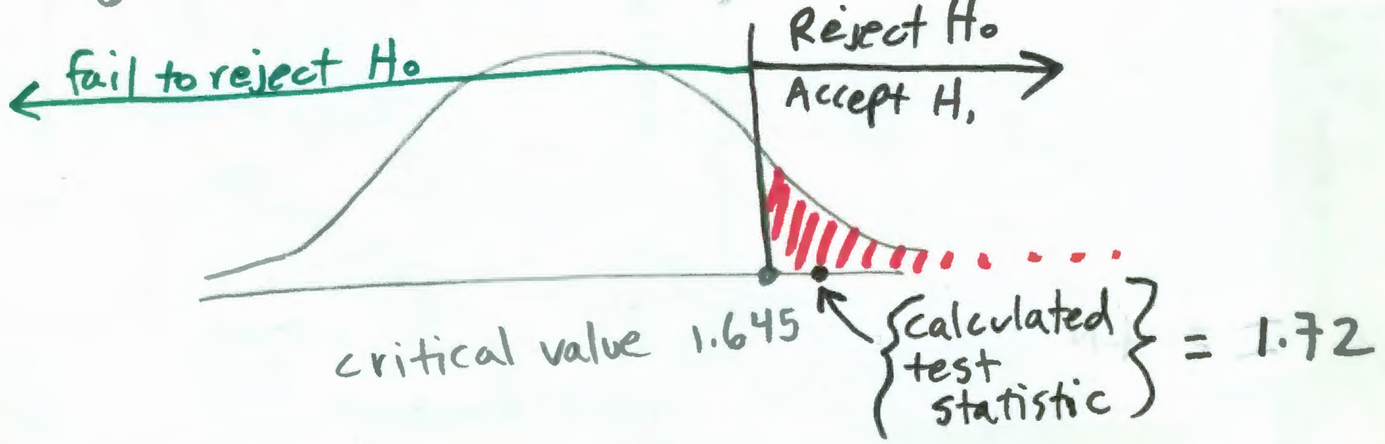
If our test statistic is greater than or equal to 1.6448, we reject Ho and accept Ha.

(step 5) conclude with critical value & Rejection Rule

$$\left\{\begin{array}{l}\text{Calculated}\\ \text{test}\\ \text{statistic}\end{array}\right\} = \frac{88595 - 85000}{\frac{12549}{\sqrt{36}}} = 1.72$$

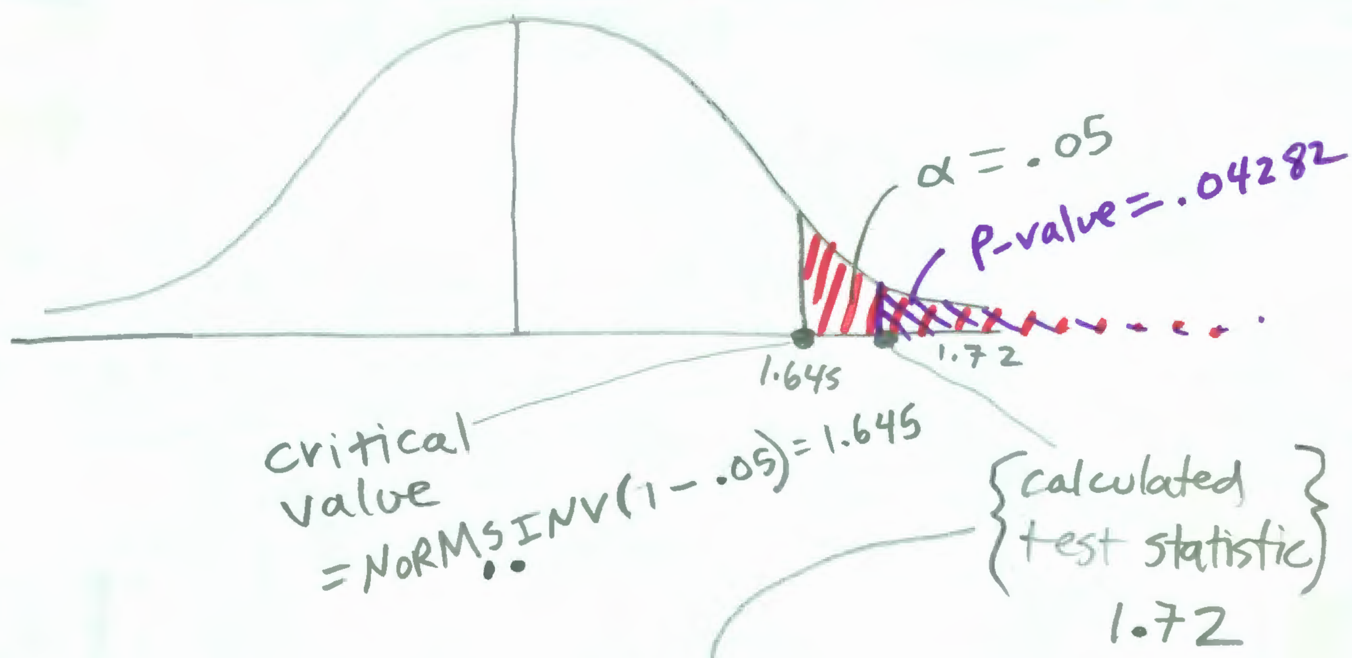
Make Decision:

Because our calculated test statistic is greater than 1.645, we reject H_0 and accept H_1. It is reasonable to assume that the mean salary for real estate agents is greater than $85000.



Based on the statistical evidence our $\overline{X}$ of 88,595 is statistically significant & provides good evidence that the mean salary for Realtors is more than 85,000

Step 4 & 5 for p-value



p-value = 1 − NORMSDIST(1.72) = .04282

Because the p-value is less than alpha (.04282 ≤ .05), we reject H_0 & accept H_1. It is reasonable to assume that the mean salary for real estate agents is greater than $85,000

Summary for Real Estate Example:

$\mu = \mu_{\bar{x}} = \mu_0 = 85000$

$\bar{x}$ $\bar{x} = 88,595$

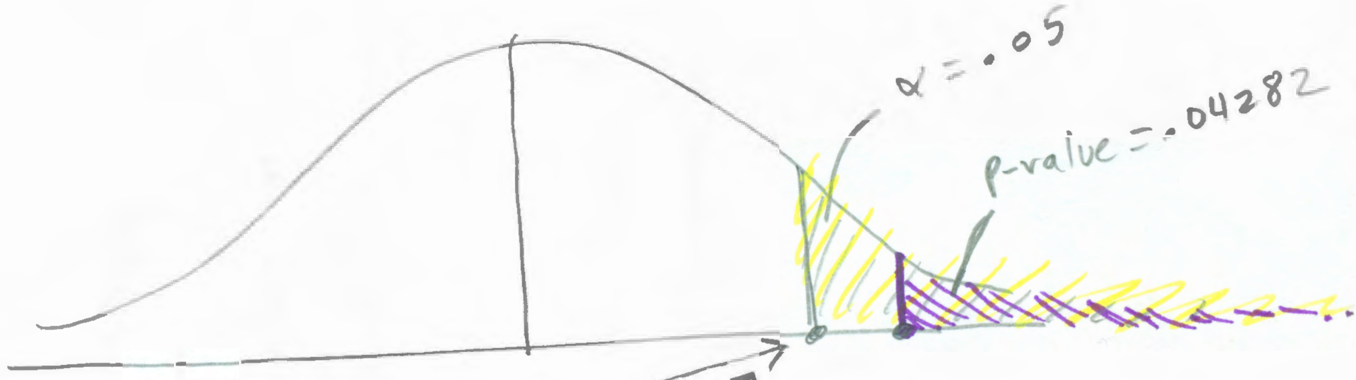
size $n = 36$

sigma $\sigma = \$12,549$

Standard Error $= \sigma_{\bar{x}} = 12549/\sqrt{36} = \$2,091.50$

alpha $\alpha = .05$

test statistic $= \dfrac{88595 - 85000}{2091.5} = 1.72$



Critical Value

Dividing point between the region where the Null Hypothesis is rejected and the region where it is not rejected

upper test

$=$ NORM.S.INV $(1 - \alpha)$

$=$ NORM.S.INV $(1 - .05) = 1.6448$

critical value $= 1.6448$

p-value

Probability of getting test statistic or more $= 1 -$ NORM.S.DIST (z)

p-value $= 1 -$ NORM.DIST $(1.72) = .04282$

concluding :

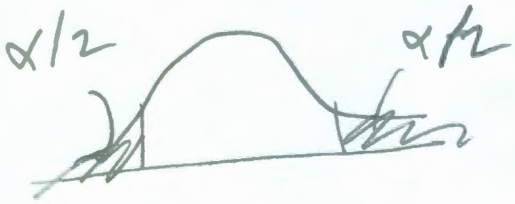
use z or t to compare to critical value

use p-value to compare to alpha α

Critical value

t or $Z \geq$ critical value, Reject H_o, Accept H_a

t or $Z \leq$ critical value, Reject H_o, Accept H_a



$-t$, $-Z \leq$ critical value $\leq t$, Z, Fail to Reject H_o

---

p-value

p-value $\leq \alpha$, Reject H_o, Accept H_a

Confidence Interval Hypothesis Testing

If:

$H_0 : \mu = \mu_0$

$H_a : \mu <> \mu_0$

Then:

$\neq$ = Not = <>

↑ Excel Symbol for "Not" or "Not Equal"

μ_0 = hypothesized population mean

① Select a simple random sample from the population and use the value of the sample mean $\bar{X}$ to develop a confidence interval for the population mean μ.

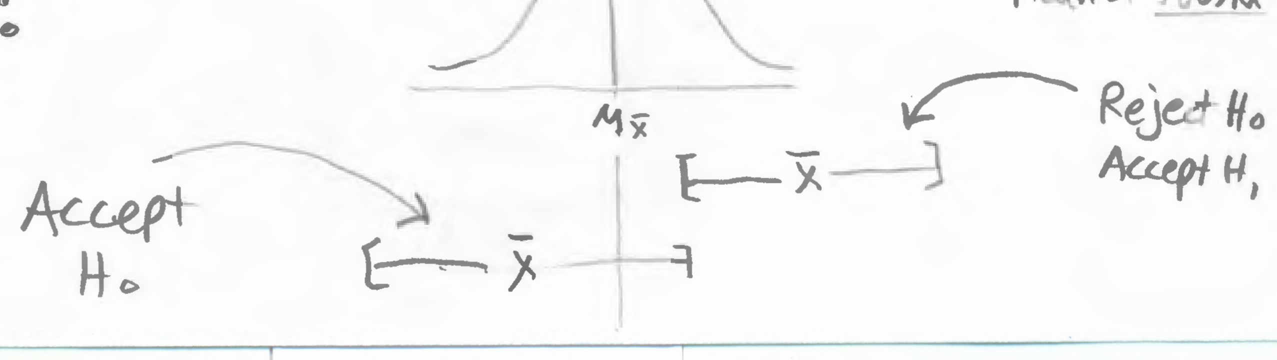
$$\bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$\bar{x}$ = sample mean
$Z_{\alpha/2}$ = upper Z
σ = pop S.D.
n = sample size

② If the confidence interval contains the hypothesized value (μ_0) μ_0, do not reject H_0, otherwise, Reject H_0 (Reject H_0 if μ_0 is one of the end points)

$\mu = \mu_{\bar{x}}$ = pop mean = mean of SDOSM

Example:



Video 53

p 40

Step 3

Ho :

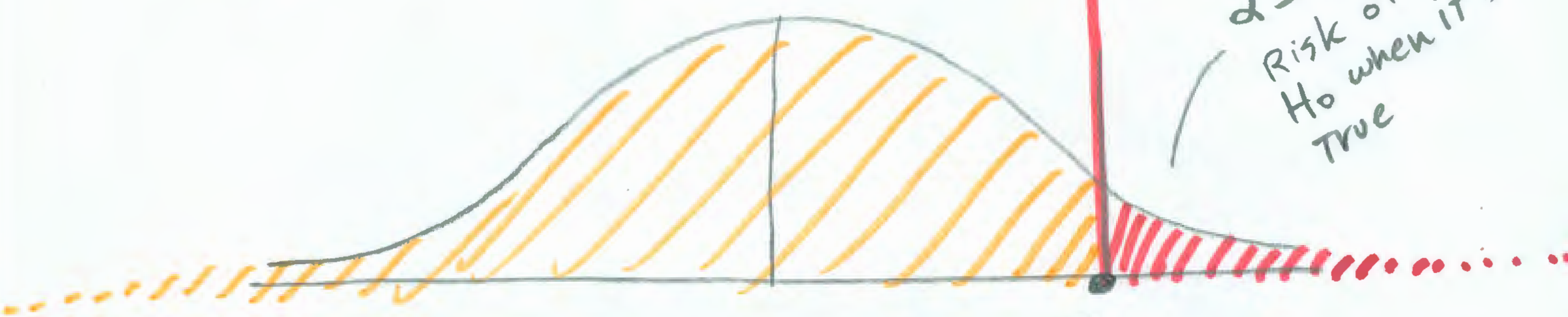
Ha :

Reject Ho

Accept Ha

Fail to Reject Ho

$\alpha = 0.05 =$ Risk of Rejecting Ho when it is True



$M = 85{,}000$

α determined Hurdle. Beyond this point we will say sample error is statistically significant

Step 4

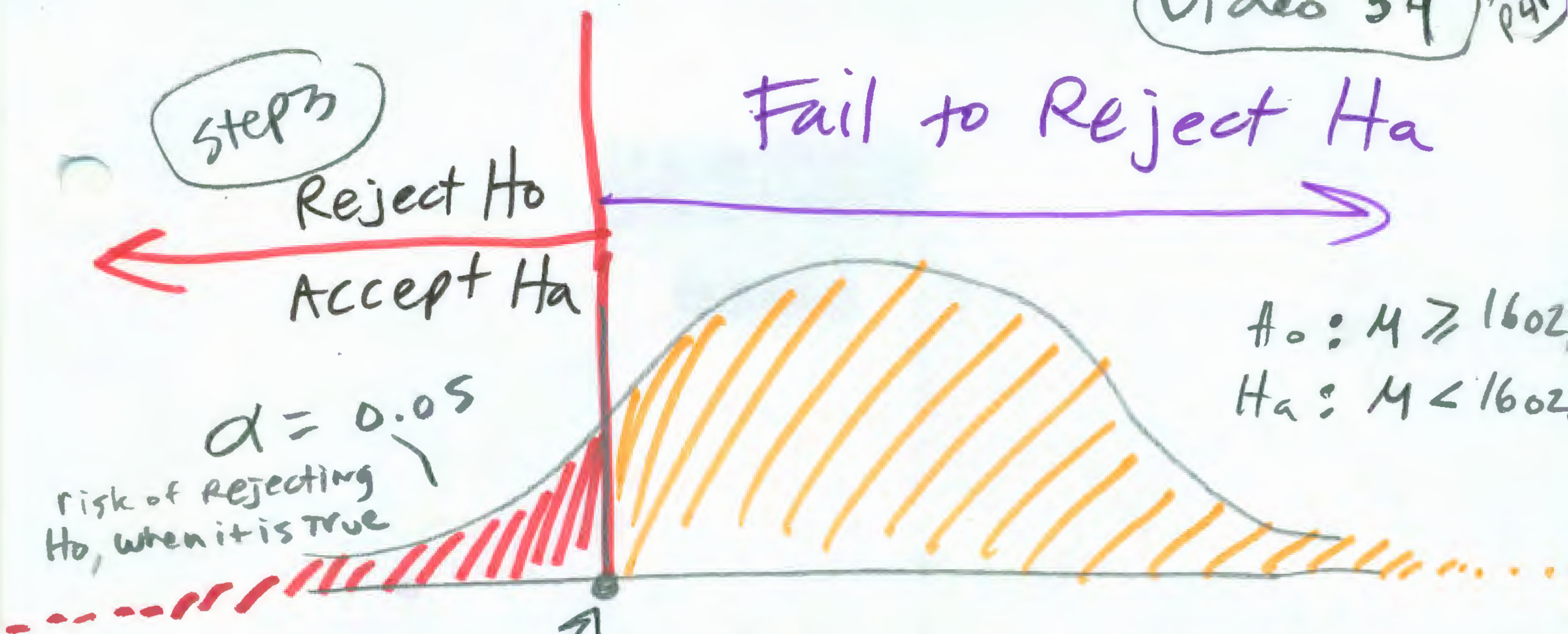
Step 5	alpha
p-value $0.043 \leq 0.05$ so Reject Ho, Accept Ha	
Test statistic $1.72 \geq 1.645$ C.V. so Reject Ho, Accept Ha	

p-value = probability of getting Z of 1.72 or more $= 0.043$

critical value = 1.645

Z Test statistic = # standard Errors = 1.72
(# standard Deviations for sampling Dist. $\bar{x}$)

step 3

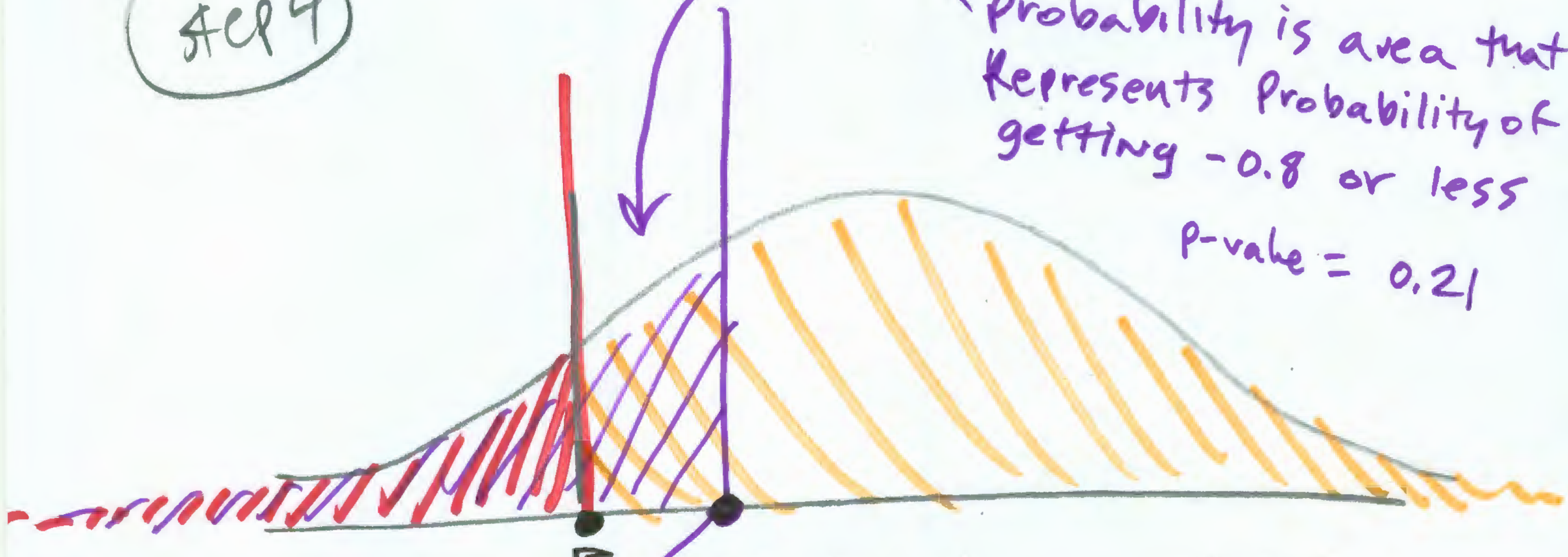


α determine Hurdle. Beyond this Point we will say sample error is statistically significant

step 4

p-value = Probability is area that Represents Probability of getting -0.8 or less

p-value = 0.21



critical value = -1.645

z Test Statistic = -0.8

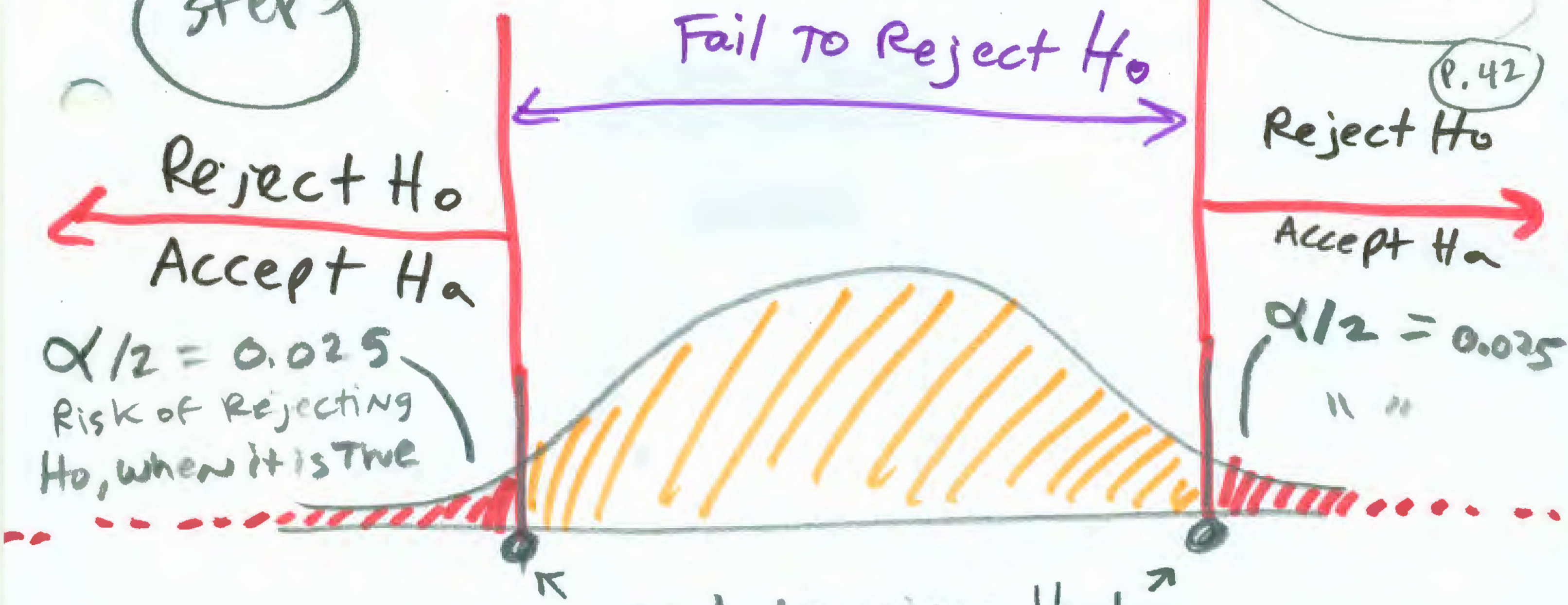
step 5

Because -0.8 > , we fail to Reject Ho

Because 0.21 > 0.05, we fail to Reject Ho

step 3

video 55 (P 42)



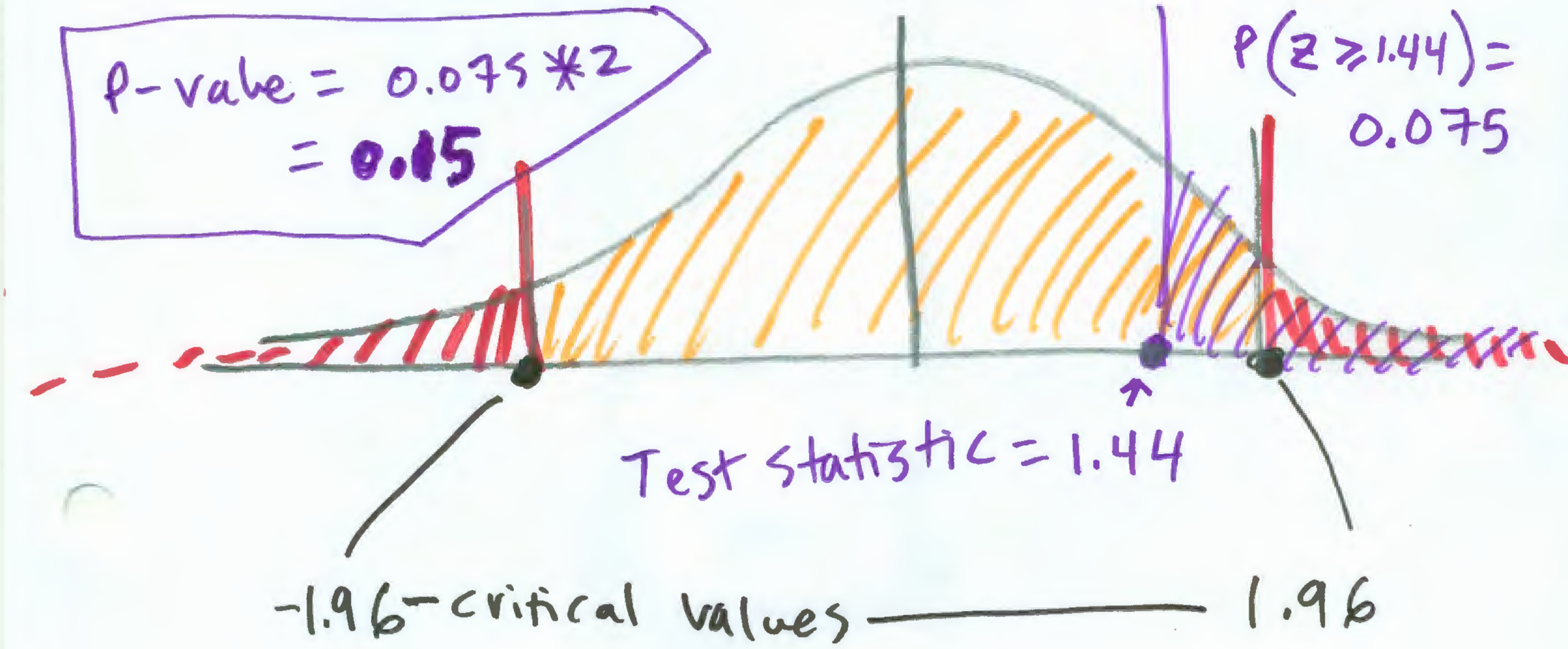
α determines Hurdes Point at which we say sample error is statistically significant

p-value for Two-Tail = probability of 1.44 or more TIMES 2 !!!

*supposed to be symmetric

$\boxed{\text{p-vale} = 0.075 * 2 = \mathbf{0.15}}$

$P(z \geq 1.44) = 0.075$



Step 3

$H_0: \mu \le 250$

$H_a: \mu > 250$

Fail to Reject H_0

Reject H_0

Accept H_a

p. 43

$\alpha = 0.01$

risk of Reject H_0, when it is True

t

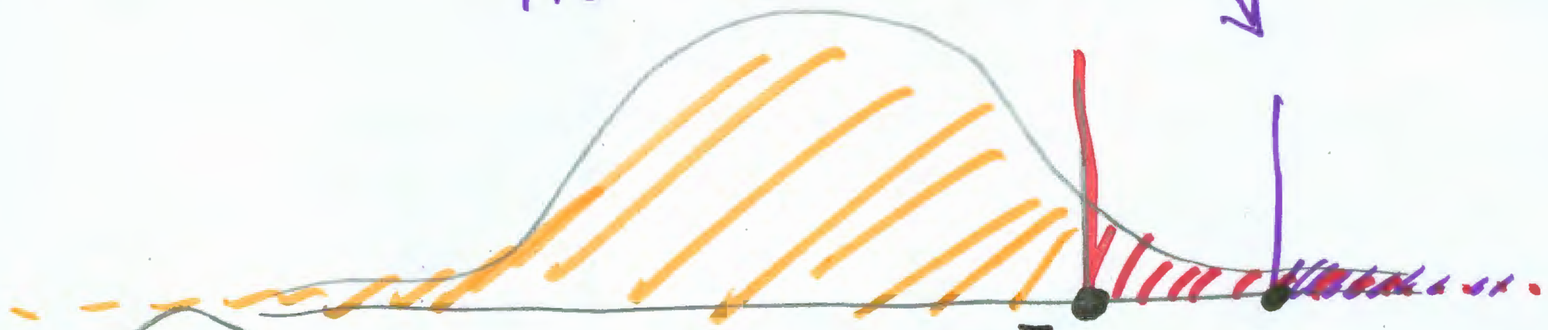
α determines critical value

cv is Hurdle

video 57

Step 4

very small p-value means very strong evidence against H_0



t

critical value = 2.62

t test statistic = 3.87

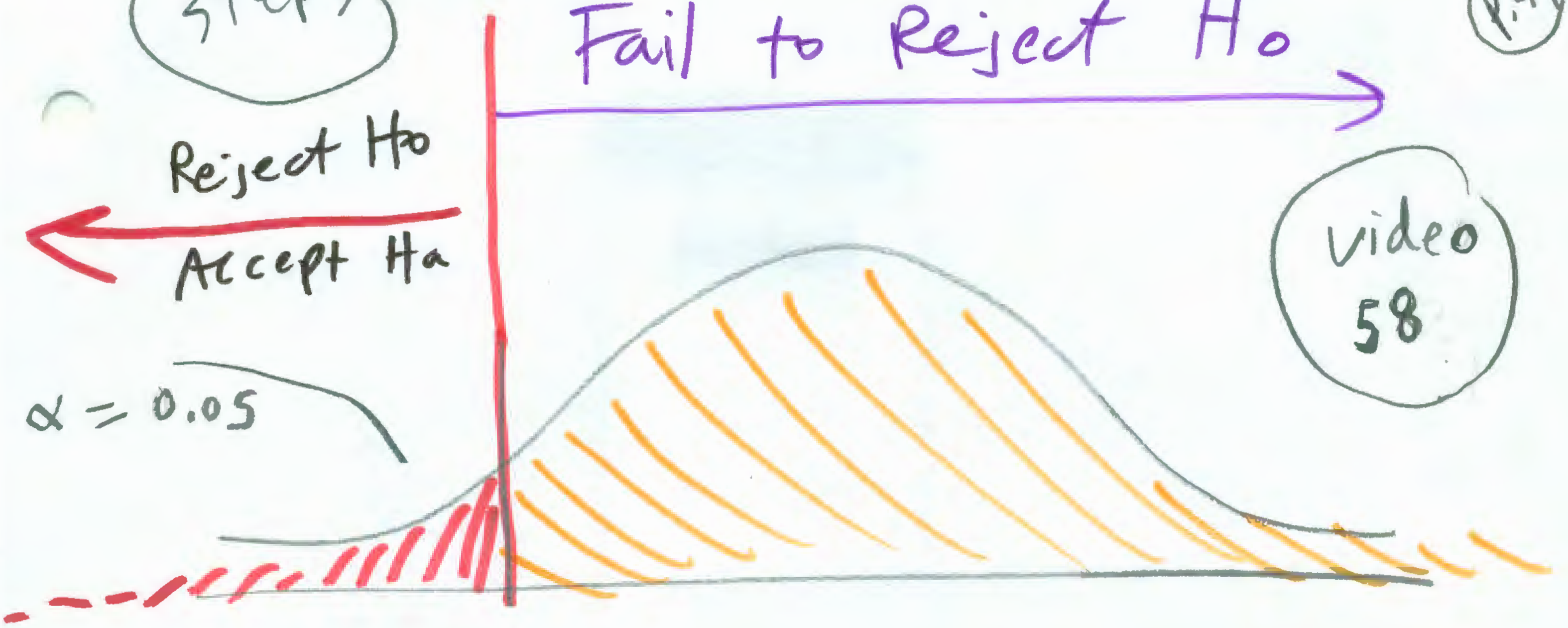
p-value = probability of getting 3.87 t or more

p-value = 0.0008 = very strong evidence

Step 3

p.44

video 58



Step 4

p-value = Probability of getting −0.944 or Less

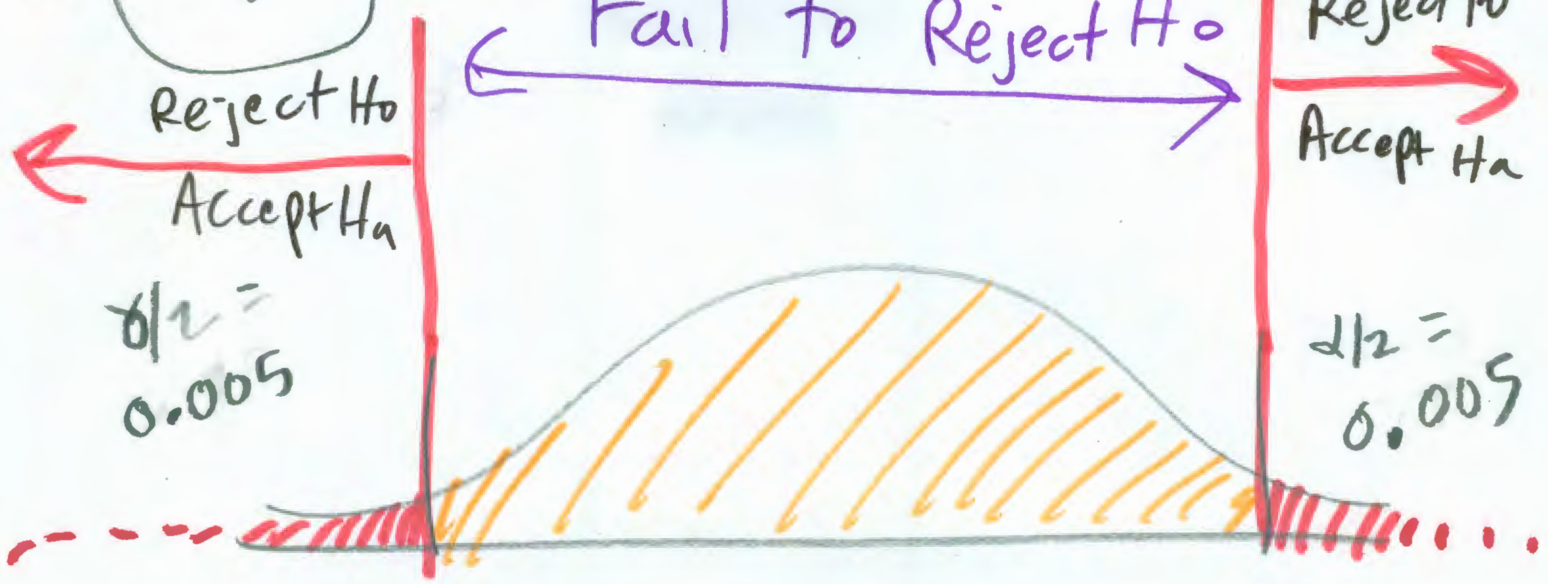
Teststatistic = −.944

p-value = 0.175

Critical value = −1.68

$H_0: \mu = 9500$
$H_a: \mu <> 9500$

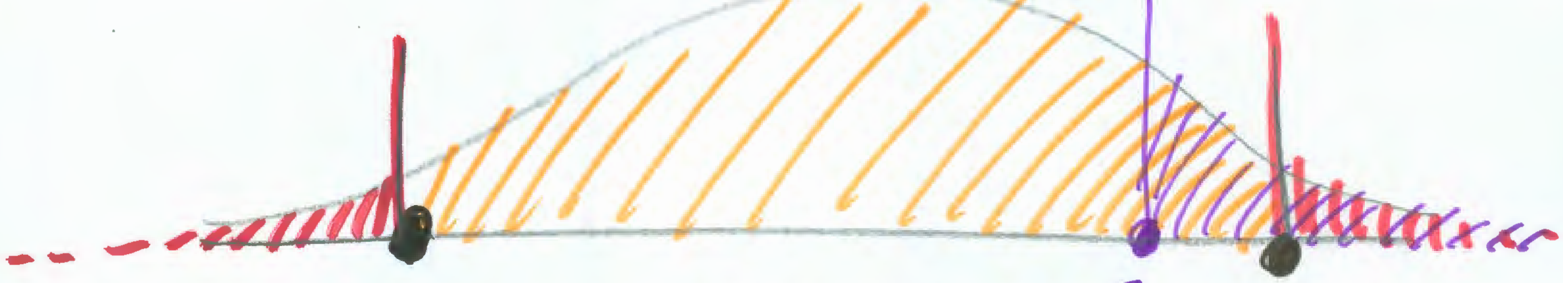
step 3



video 59

step 4

$\frac{1}{2}$ p-value = Probability of getting 1.52 or more
MUST DOUBLE
0.0663



test statistic = 1.52

-1.99

+/- critical value = 1.99

P-value = 0.0663 * 2 = 0.1326

③ Decision Making → Choose between 2 things → H_0 or H_a

Example:

Should we accept box of 20 meter boomerangs?

(if they fly too short, can't be used in competition)
(if they fly too long, times will not be fast enough.)

Choice #1 → $H_0 : \mu = 20$ meters → accept Box

Choice # 2 → H_a or $H_1 : \mu \neq 20$ meters → reject Box