Introduction To Probability Chapter 4 Notes by excelisfun

Probability

Probability = likelihood = chance = possibility

Probability is a numerical measure, a number between 0 and 1 (inclusive), that indicates the likelihood that an event will occur in the unknown future.

Probability is never known with certainty. It is only an estimate.

Probability is an estimate of an event that may occur in the future.

Probability is never negative.

Probability is never greater than 1.

A percentage change amount is not probability. Remember, you can have an increase in sales of 110% (>1) or a decrease in sales of -25% (<0), but those are NOT probabilities.

Probability represents parts out of 100, where you can have 0 to 100 parts out of 100. If the probability of a sale for any one sales call is 0.20, this means that in a random test you would expect to make 20 sales for every 100 sales calls.

Examples:

1) Probability that you will roll a 6 with a die = P(roll six) = 1/6 = 0.1667 = 16.67%
2) Probability that a randomly selected student in my class will earn an A = P(Earn A) = 0.10 = 10%
3) On Jan. 25, 2022, a Casino estimated probability that the KC Chiefs would win super bowl = P(win) = 0.43 =43%
4) On Jan 31, 2022, the probability that the KC Chiefs would win super bowl = P(win) = 0 = 0%
5) The probability that it will rain in Seattle next year = approximately 1 = 100%

Methods for estimating probability:

Classical Probability = All outcomes equally likely.
Example: probability of rolling a 3 with one die = 1/6 = 0.1666.

Relative Frequency Probability = Use past data to create relative frequency distribution.
Example: probability of getting an A in a class based on past data = 5/50 = 1/10 = 0.10

Subjective Probability = Expert judgement because outcomes are not equally likely and there is little past data.
Example: Casino estimates that the probability that KC Chiefs will win Super Bowl = 0.43

Random Experiment

A process that generates well defined **Experimental Outcomes (Sample Points)**.

On any single repetition or trial or step of the experiment, one and only one of the possible experimental outcomes (sample points) can occur.

The experimental outcome that occurs on any trial is determined solely by change.

Sample Space for a random experiment

A set of all experimental outcomes for a random experiment.

It is not always possible to list all experimental outcomes.

Examples of **1-step** random experiment:

Random Experiment:	**Sample Space:**
1) Roll a die	1,2,3,4,5,6
2) Select a product for inspection	Defect, Not Defect
3) Play Super Bowl	Win, Lose
4) Play NFL game	Win, Lose, Tie

Examples of **multi-step** random experiment:

Random Experiment:

1) Flip coin two times
2) Roll two die (dice)
3) 2-step building zoning process

Sample Space:

(H,H), (H,T), (T,H), (T,T)

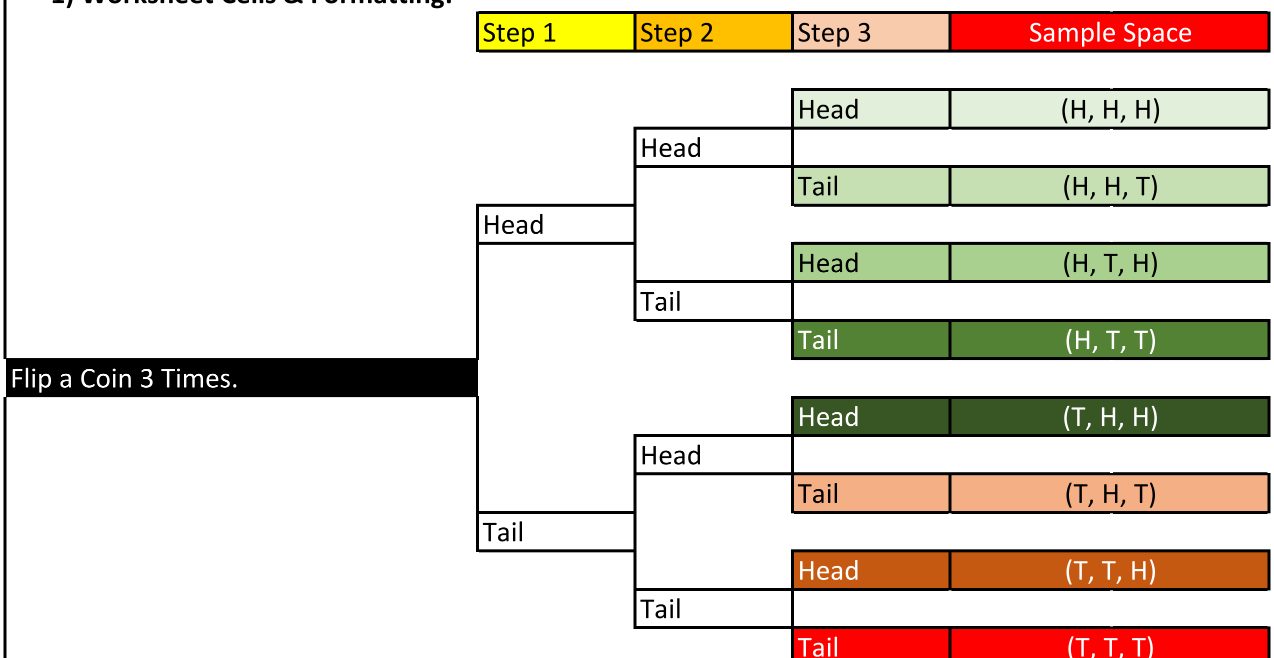
(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

(Positive Recommendation, Approve), (Positive Recommendation, Disapprove), (Negative Recommendation, Approve), (Negative Recommendation, Disapprove)

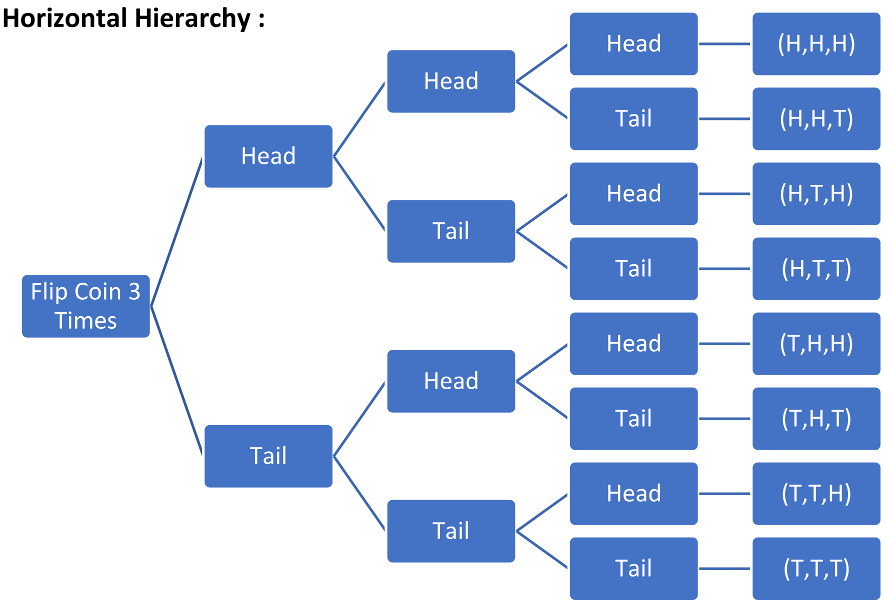
Use **Tree Diagram** to visualize Sample Space and show all experimental outcomes (sample points):

Three Examples for random experiment flipping a coin three times:

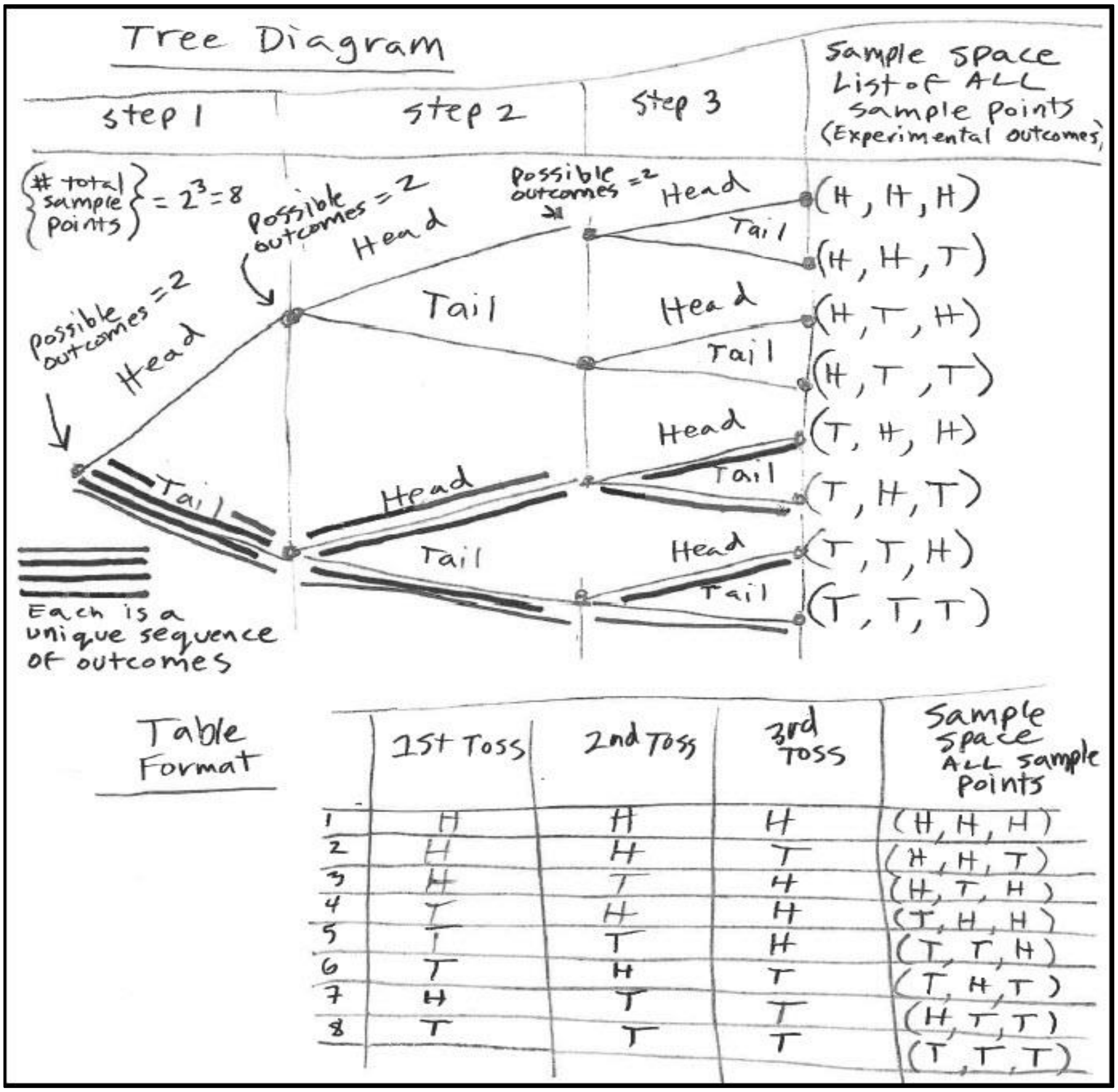
1) Worksheet Cells & Formatting:



2) Excel Smart Art Horizontal Hierarchy :



3) Drawing on Paper:



Use **Table Format** to visualize Sample Space and show all experimental outcomes (sample points):

<u>Example of table to visualize all sample points for a two-step random experiment of throwing two die:</u>

Die1/Die2	1	2	3	4	5	6
1	(1,1) = 2	(1,2) = 3	(1,3) = 4	(1,4) = 5	(1,5) = 6	(1,6) = 7
2	(2,1) = 3	(2,2) = 4	(2,3) = 5	(2,4) = 6	(2,5) = 7	(2,6) = 8
3	(3,1) = 4	(3,2) = 5	(3,3) = 6	(3,4) = 7	(3,5) = 8	(3,6) = 9
4	(4,1) = 5	(4,2) = 6	(4,3) = 7	(4,4) = 8	(4,5) = 9	(4,6) = 10
5	(5,1) = 6	(5,2) = 7	(5,3) = 8	(5,4) = 9	(5,5) = 10	(5,6) = 11
6	(6,1) = 7	(6,2) = 8	(6,3) = 9	(6,4) = 10	(6,5) = 11	(6,6) = 12

Die 1 formula: =SEQUENCE(6)
Die 2 formula: =SEQUENCE(,6)
Inside formula: ="("&C125#&","&D124#&") = "&C125#+D124#

Counting Rule for Multi-Step Random Experiment

Total number of experimental outcomes (sample points) = size of sample space = $\mathbf{n_1 * n_2 * \ldots n_k}$

k = Number of steps or trials in the random experiment

n_1 = number of possible outcomes in step 1

n_2 = number of possible outcomes in step 2

n_k = number of possible outcomes in last step

Examples:

1) Roll two die. k = 2, n_1 = 6, n_2 = 6. Total experiments outcomes = 6*6 = 36

2) Determine lock code with 3 spots and 10 digits. What are the number of total experimental outcomes (total possible lock codes)?

Number slots for lock code:	k	3	
Digits: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10	n_1	10	
Digits: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10	n_2	10	
Digits: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10	n_3	10	
Total lock possibilities:		1000	10*10*10 or 10^3

3) Deli offers 2 rolls, 4 meats and 3 cheeses. k = 3, n_1 = 2, n_2 = 4, n_3 = 3. What are the number of total experimental outcomes (total number of different sandwiches) if you get one of each?

# steps in building sandwich:	k	3	
Number rolls	n_1	2	
Number meats	n_2	4	
	n_3	3	
Total lock possibilities:		24	2*4*3

Sometimes we have to decide on sample space with combinations or permutations, where you select n objects from a set of N objects.

Example of the difference between a Combination and a Permutation:

If we select the letters A, B, C, one after the other and you cannot repeat (example: A, A, A not allowed)

	Permutations	Combinations
	Order Matters	Order Does Not Matter
	A, B, C	A, B, C
	A, C, B	
	B, A, C	
	B, C, A	
	C, A, B	
	C, B, C	
Total	**6**	**1**

* You can pick the letters in any order, but just one time.

Counting Rule for Combinations, Order Does Not Matter

When experiment involves selecting n objects from a set of N objects, and the order of the items is not considered, like: (2,1) is the same as (1,2), then:

Total number of experimental outcomes (sample points) = size of sample space =

$$\# \, Combinations = C_n^N = \binom{N}{n} = \frac{N!}{n!(N-n)!}$$

N = count of all objects (population)	7	Population Size
n = count of objects selected (sample size)	3	Sample Size
In Excel use function: COMBIN(N,n)	35	Total combinations
	=COMBIN(7,3)	

Note: Factorial (from your algebra class) is represented by ! Example: 5! = 5*4*3*2*1 = 120, and: 0! = 1.

Examples:

1) Find all possible combinations of sample size 3 from a set of 7 numbers = 7!/(3!*(7-3)!) = (7*6*5*4*3*2*1)/(3*2*1*(4*3*2*1)) = (7*6*5)/(3*2*1*) = 7*5 = 35

2) Find all combinations a for a basketball team that has 13 players and 5 can play in a game, assuming a player can play any position = 13!/(5!*(13-5) = (13*12*11*10*9)/(5*4*3*2*1) = (13*3*11*2*3)/2 = 1287

Counting Rule for Permutations, Order Matters

When experiment involves selecting n objects from a set of N objects, and the order of the items is considered, like: (2,1) is ***different*** than (1,2), then:

Total number of experimental outcomes (sample points) = size of sample space =

$$\# \, Permutations = P_n^N = \mathrm{n!} \binom{N}{n} = \frac{N!}{(N-n)!}$$

N = count of all objects (population)	5	Population Size
n = count of objects selected (sample size)	5	Sample Size
In Excel use function: PERMUT(N,n)	120	Total permutations
	=PERMUT(5,5)	

Examples:

1) Find total possible arrangements for 5 employees in 5 different offices = 5!/(5-5)! = (5*4*3*2*1)/0! = 120/1 = 120

2) Find total possible arrangements for 5 employees in 3 different offices = 5!/(5-3)! = (5*4*3*2*1)/2! = 120/2 = 60

Good site for other types of combinations and permutations, and for a description of why these formulas are valid:
https://www.mathsisfun.com/combinatorics/combinations-permutations.html

Basic Requirements for Assigning Probabilities

1] The probability for each experimental outcome (sample point) must be between 0 and 1, inclusive.

$0 <= P(E_i) <= 1$ for all i, where: Ei = ith experimental outcome and P(Ei) = Probability.

2] The sum of the probabilities for all experimental outcomes (sample points) from the sample space must be equal to 1.

$P(E_1) + P(E_2) + ... + P(E_n) = 1$, where there are n experimental outcomes.

Event

A collection of one or more experimental outcomes (sample points).

Examples:

1) The event roll a 7 with dice has the following sample points: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1).

2) The event get one or more tails in two flips of a coin has the following sample points:
(T,T), (H,T), (T,H)

3) The event sold a Quad from a list of products sold: Quad,Carlota,Quad,Sunshine, has the following sample points:
(Quad,Quad).

Note: Sample points and events provide the foundation for the study of probability.

Probability of an Event

The probability of an event is equal to the sum of the probabilities of the experimental outcomes (sample points) in the event.

Examples:

1) Event = Roll a 7 with dice

Sample points = (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)

Probability = P(Roll 7) = 1/36+1/36+1/36+1/36+1/36+1/36 = 6/36 = 1/6 = 0.1667

All sample points for experiment "roll dice":

Die1/Die2	**1**	**2**	**3**	**4**	**5**	**6**
1	(1,1) = 2	(1,2) = 3	(1,3) = 4	(1,4) = 5	(1,5) = 6	(1,6) = 7
2	(2,1) = 3	(2,2) = 4	(2,3) = 5	(2,4) = 6	(2,5) = 7	(2,6) = 8
3	(3,1) = 4	(3,2) = 5	(3,3) = 6	(3,4) = 7	(3,5) = 8	(3,6) = 9
4	(4,1) = 5	(4,2) = 6	(4,3) = 7	(4,4) = 8	(4,5) = 9	(4,6) = 10
5	(5,1) = 6	(5,2) = 7	(5,3) = 8	(5,4) = 9	(5,5) = 10	(5,6) = 11
6	(6,1) = 7	(6,2) = 8	(6,3) = 9	(6,4) = 10	(6,5) = 11	(6,6) = 12

All probabilities for experiment "roll dice":

Die1/Die2	**1**	**2**	**3**	**4**	**5**	**6**
1	1/36	1/36	1/36	1/36	1/36	1/36
2	1/36	1/36	1/36	1/36	1/36	1/36
3	1/36	1/36	1/36	1/36	1/36	1/36
4	1/36	1/36	1/36	1/36	1/36	1/36
5	1/36	1/36	1/36	1/36	1/36	1/36
6	1/36	1/36	1/36	1/36	1/36	1/36
					Total	1

Probability Requirement #1 is met: each 1/36 probability is between 0 and 1.

Probability Requirement #2 is met: sum of all probability equals 1: 1/36*36 = 1.

2) Event = Get 2 tails in three flips of a coin

Sample points = (H, T, T), (T, H, T), (T, T, H)

Probability = P(2T in 3 Tries) = 1/8+1/8+1/8 = 3/8 = 0.375

All probabilities for experiment "flip fair coin three times":

Step 1	Step 2	Step 3	Sample	P(SP)
H	H	H	(H, H, H)	1/8
H	H	T	(H, H, T)	1/8
H	T	H	(H, T, H)	1/8
T	H	H	(T, H, H)	1/8
H	T	T	(H, T, T)	1/8
T	H	T	(T, H, T)	1/8
T	T	H	(T, T, H)	1/8
T	T	T	(T, T, T)	1/8
			Total	**1**

Probability Requirement #1 is met: each 1/8 probability is between 0 and 1.

Probability Requirement #2 is met: sum of all probability equals 1: 1/8*8 = 1.

3) Event = Use 2 or more banquet rooms at Isaac's Italian Restaurant on a weekend day.

"Sample points" from pre-made frequency distribution = 2 rooms used, 3 rooms used, 4 rooms used.

Probability = P(Rooms Used>=2) = 0.43 + 0.27 = 0.08 = 0.78, or, (45 + 28 + 8)/104 = 81/104 = 0.78

Summary of all 104 sample points & probabilities into a frequency distribution:

	# Rooms Used in Day (x)	Frequency	% Frequency or P(x)
	0	2	2%
	1	21	20%
45 sample points @ 1/104 probability each ➞	2	45	43%
28 sample points @ 1/104 probability each ➞	3	28	27%
8 sample points @ 1/104 probability each ➞	4	8	8%
	Total	**104**	**100.0%**

Probability Requirement #1 is met: each probability is between 0 and 1.

Probability Requirement #2 is met: sum of all probability equals 1: 2%+20%+43%+27%+8% = 1

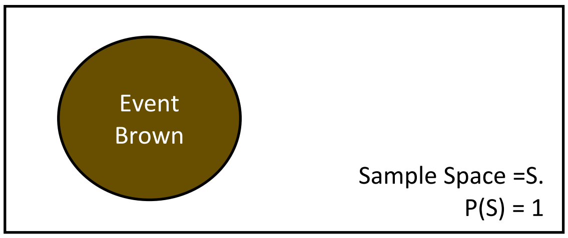
Venn Diagram

Diagram to show relationship between different events and the sample space for an experiment.
Rectangle = sample space = all sample points possible.
Circle = Event (one or more sample points).

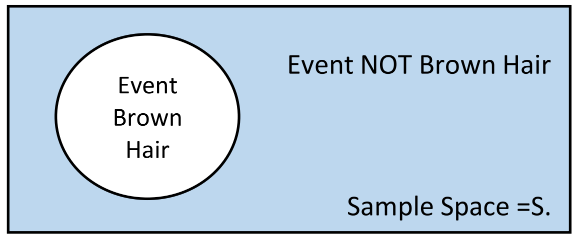
Name	Hair Color	Eye Color
Sioux	Brown	Hazel
Chin	Black	Blue
Shelia	Black	Brown
Gigi	Brown	Hazel
Tyrone	Black	Brown
Lin	Blond	Brown
Ducky	Brown	Brown
Kip	Brown	Hazel
Linda	Red	Blue
Jo	Pink	Green
Total Records:		10

Event **Black Hair**

Event **Brown Hair**



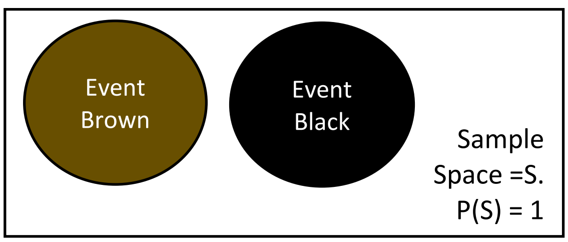
Event **NOT Brown Hair**



Complement Rule

Given an Event A, the complement of A contains all sample points that are NOT in A.

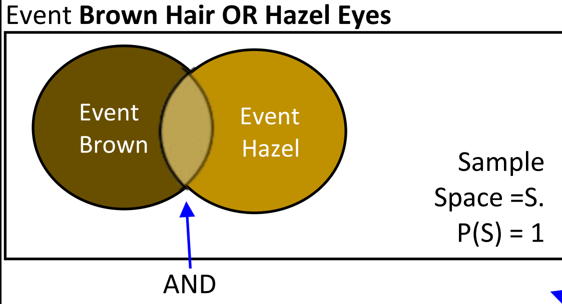
Event **Brown Hair OR Black Hair**



Mutually Exclusive Events

The two events have no sample points in common.
No intersection. Not overlap.
If one mutually exclusive event is known to occur, the other event cannot occur and its probability is reduced to 0.

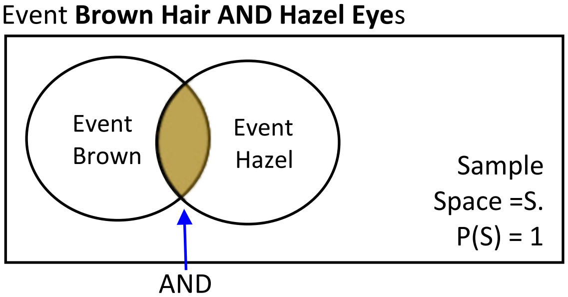
Event **Brown Hair OR Hazel Eyes**



Union of Two Events, OR Logical Test

The union of A and B is the event that contains all sample points belonging to A or B or both.
If you are counting, you must subtract the AND section to avoid double counting.

Event **Brown Hair AND Hazel Eyes**



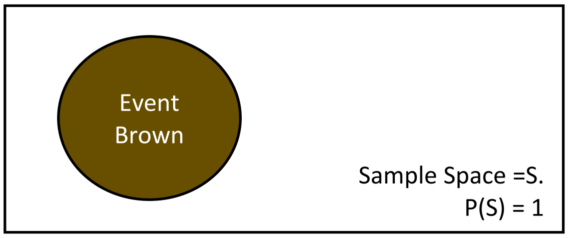
Intersection of Two Events, AND Logical Test

The intersection of A and B is the event that contains sample points that belonging to both A and B. There is overlap.
If you are counting, you only count what is in the AND section.

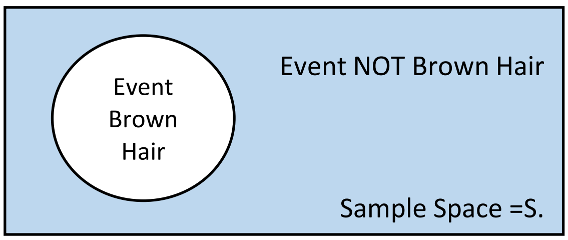
Logical Tests Used for Counting Sample Points

Event **Black Hair**

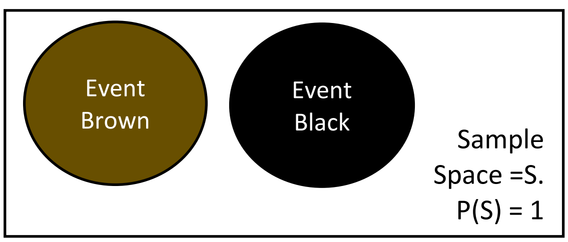
Event **Brown Hair**



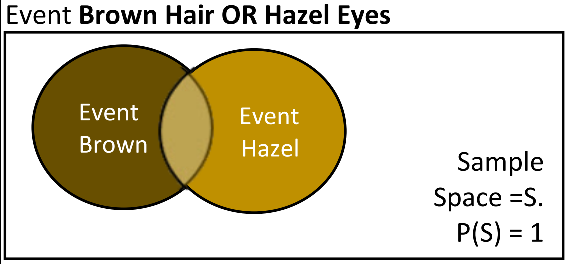
Event **NOT Brown Hair**



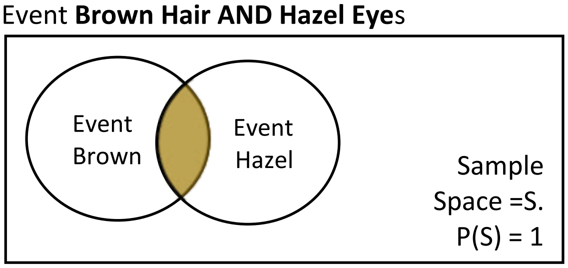
Event **Brown Hair OR Black Hair**



Event **Brown Hair OR Hazel Eyes**



Event **Brown Hair AND Hazel Eyes**



Name	HairColor	EyeColor
Sioux	Brown	Hazel
Chin	Black	Blue
Shelia	Black	Brown
Gigi	Brown	Hazel
Tyrone	Black	Brown
Lin	Blond	Brown
Ducky	Brown	Brown
Kip	Brown	Hazel
Linda	Red	Blue
Jo	Pink	Green
Total Records:		10

1) Single Condition Logical Test: Equal to Brown Hair

Condition	Brown	
Count	4	=COUNTIFS(HairColor,G354)

P(Brown)	0.4

2) Not Logical Test: NOT Brown Hair

Condition	Brown	
Count	6	=COUNTIFS(HairColor,"<>"&G359)

P(Brown)	0.6

Complement Rule: If P(B), then P(Not B) = 1 - P(B)

P(Brown)	0.6	1-0.4

3a) OR Logical Test for Mutually Exclusive Events (One Column):

Brown Hair OR Black Hair

Condition	Brown
Condition	Black
Count	7

=ROWS(FILTER(HairColor,(HairColor=G368)+(HairColor=G369)))

Count	7

=COUNTIFS(HairColor,G368)+COUNTIFS(HairColor,G369)

3b) OR Logical Test for Events that are not Mutually Exclusive

Brown Hair OR Hazel Eyes **(Two Columns):**

Condition	Brown	Hair
Condition	Hazel	Eyes
Count	4	

=ROWS(FILTER(H341:I350,(HairColor=G377)+(EyeColor=G378)))

Count	4

=COUNTIFS(HairColor,G377)+COUNTIFS(EyeColor,G378)-COUNTIFS(HairColor,G377,EyeColor,G378)

4) AND Logical Test: Brown Hair AND Hazel Eyes

Condition	Brown	Hair
Condition	Hazel	Eyes
Count	3	

=COUNTIFS(HairColor,G385,EyeColor,G386)

Logical Tests Used for Filtering Sample Points

We can use the **FILTER array function** to filter a data set in order to see specified sample points.

FILTER(ArrayToFilter,LogicalTestArray)

ArrayToFilter & LogicalTestArray must be same size.

OR Logical Test uses + in direct array operation.

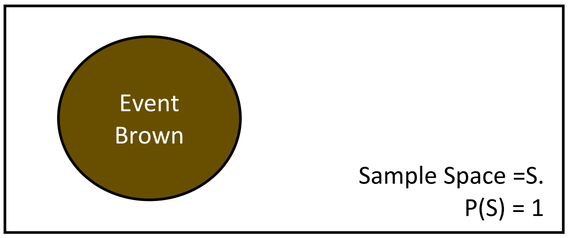
AND Logical Test uses * in direct array operation.

TRUE = TRUE or any non-zero number.

FALSE = FALSE or 0.

Name	HairColor	EyeColor
Sioux	Brown	Hazel
Chin	Black	Blue
Shelia	Black	Brown
Gigi	Brown	Hazel
Tyrone	Black	Brown
Lin	Blond	Brown
Ducky	Brown	Brown
Kip	Brown	Hazel
Linda	Red	Blue
Jo	Pink	Green
Total Records:		10

Event **Brown Hair**



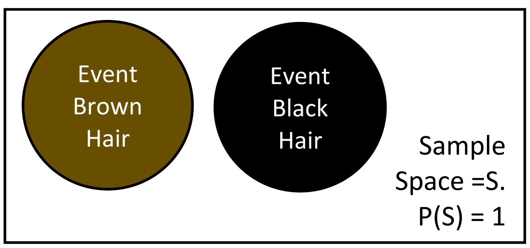
1) Single Condition Logical Test: Equal to Brown Hair

Condition	Brown	Matching Rows:

=FILTER(HairColor,HairColor=G405)

Matching Rows:
Brown
Brown
Brown
Brown

Event **Brown Hair OR Black Hair**



3a) OR Logical Test for Mutually Exclusive Events (One Column):

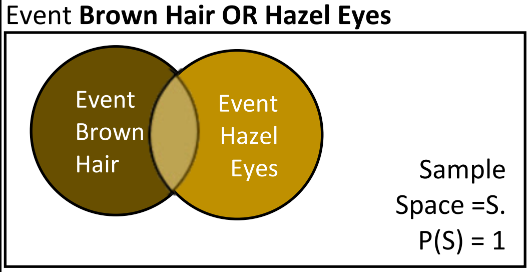
Brown Hair OR Black Hair

Condition	
Condition	Brown
Condition	Black

=FILTER(HairColor,(HairColor=G413)+(HairColor=G414))

Matching Rows:
Brown
Black
Black
Brown
Black
Brown
Brown

Event **Brown Hair OR Hazel Eyes**



3b) OR Logical Test for Events that are not Mutually Exclusive (Two Columns):

Brown Hair OR Hazel Eyes

Condition		
Condition	Brown	Hair
Condition	Hazel	Eyes

=FILTER(H392:I401,(HairColor=G425)+(EyeColor=G426))

Matching Rows:	
Brown	Hazel
Brown	Hazel
Brown	Brown
Brown	Hazel

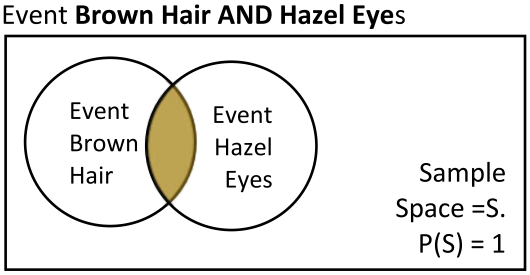
4) AND Logical Test: Brown Hair AND Hazel Eyes

Condition		
Condition	Brown	Hair
Condition	Hazel	Eyes

=FILTER(H392:I401,(HairColor=G432)*(EyeColor=G433))

Matching Rows:	
Brown	Hazel
Brown	Hazel
Brown	Hazel

Event **Brown Hair AND Hazel Eyes**



FILTER Function

The FILTER array function allows you to filter a set of values to show only that values that meet a logical test. The **array** argument contains the values that you want to filter. The **include** argument requires an array of TRUE and FALSE values (same dimension as array argument values) to indicate which values to keep (TRUE) and with ones to filter out (FALSE).

For an **OR Logical Test**, use addition, + operation, like: =FILTER(H54:I63,(H54:H63=G87)+(I54:I63=G88))

For an **AND Logical Test**, use multiplication, * operator, like: =FILTER(H54:I63,(H54:H63=G87)*(I54:I63=G88))

COUNTIFS function

The COUNTIFS function makes a conditional count calculation based on one or more logical tests. The **criteria_range argument** contains the full range with all the conditional items. The **criteria argument** contains the conditions for counting items from the criteria_range1 argument.

If you use a single condition like with: =COUNTIFS(H3:H12,G16), you are performing a Single Condition Logical Test.

If you use two or more conditions like with: =COUNTIFS(M31:M40,G47,N31:N40,G48), you are performing an AND Logical Test.

If you need to use a comparative operator with the condition, you must join the comparative operator to the cell with the condition, like: "<>"&G21. Example for this formula counts items that are not whatever the value in cell G21 is.

You can have up to 127 pairs of criteria_rangeN criteriaN arguments that will run an AND Logical Test to make the conditional count calculation.

Comparative Operator Note:

* When using comparative operators in functions like COUNTIFS, SUMIFS, AVERAGEIFS, MINIFS and MAXIFS, you must join the comparative operator to the cell with the condition, like: ">"&J28.
* But when you use a comparative operator in a formula that makes a direct logical test formula calculation, you do not use quotes or an ampersand (join operator), like: H54:H63=G87.

More Notes for Important Terms:

Complement Rule

Sample Space = All Sample Points. P(Sample Space) = P(S) = 1.

Given an Event A, the complement of A contains all sample points that are NOT in A

If the complement of A = A^c , then **P(A^c) = 1 - P(A)**, or, **P(Not A) = 1 - P(A)**

Mutually Exclusive

Think of it as "Dating only one person".

Two events are said to be mutually exclusive if they have no sample points in common. The intersection of the two events must contain no sample points.

Categories in a frequency distribution are mutually exclusive when each item in the sample space can fit into only one category.

Events A and B are mutually exclusive if, when one event occurs, the other cannot occur. If one mutually exclusive event is known to occur, the other event cannot occur and its probability is reduced to zero.

Two mutually exclusive events are dependent because if one event occurs, the other cannot occur.

Union of Two Events, OR Logical Test

The union of A and B is the event that contains all sample points belonging to A or B or both.

Notation for Probability is: P(A U B) = P(A OR B), where U = Union / OR.

Synonyms: OR = Union = "At least 1", "1 or more"

Intersection of Two Events, AND Logical Test

The intersection of A and B is the event that contains sample points that belonging to both A and B.

Notation for Probability is: P(A ∩ B) = P(A AND B), where ∩ = Intersection / AND.

Synonyms: AND = Intersection = Concurrent = Joint = Both.

OR Logical Test

An OR Logical Test runs two or more logical test, and requires one or more of the tests to evaluate to TRUE in order for the OR logical Test to deliver a TRUE.

For two logical tests: TRUE,TRUE = TRUE; TRUE,FALSE = TRUE; FALSE,TRUE = TRUE; FALSE,FALSE = FALSE.

When running an OR Logical Test you use the math operation: addition.

When running an OR Logical Test over a single column, the events are mutually exclusive, and therefore you do NOT need to take into account the possibility of double counting.

When running an OR Logical Test over two or more columns, the events are not necessarily mutually exclusive, and therefore you must take into account the possibility of double counting.

AND Logical Test

An AND Logical Test runs two or more logical test, and requires all tests to evaluate to TRUE in order for the AND logical Test to deliver a TRUE.

For two logical tests: TRUE,TRUE = TRUE; TRUE,FALSE = FALSE; FALSE,TRUE = FALSE; FALSE,FALSE = FALSE.

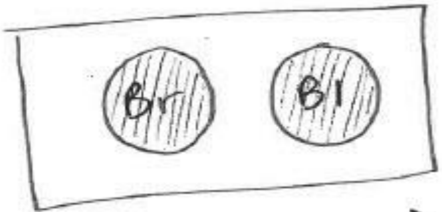
When running an AND Logical Test you use the math operation: multiplication.

Addition Law of Probability (OR Logical Test / +)

Addition Law is used to calculate the probability of the union of events (probability of an OR Logical Test).

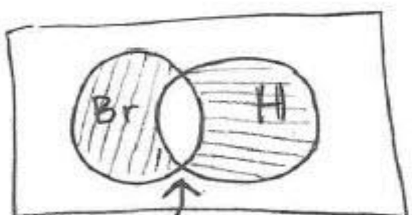
For Mutually Exclusive Events:	*Textbook notation:*
P(A OR B) = P(A) + P(B)	P(A U B) = P(A) + P(B)
For Events that are NOT Mutually Exclusive:	
P(A OR B) = P(A) + P(B) - P(A AND B)	P(A U B) = P(A) + P(B) - P(A ∩ B)

Summary of Addition Law for Probability



← {Mutually Exclusive Events}

$$P(Br \text{ OR } Bl) = P(Br) + P(Bl)$$
$$P(Br \cup Bl) = P(Br) + P(Bl)$$



← {NOT Mutually Exclusive Events}

$$P(Br \text{ OR } H) = P(Br) + P(H) - P(Br \text{ AND } H)$$
$$P(Br \cup H) = P(Br) + P(H) - P(Br \cap H)$$

{Must subtract so you DO NOT DOUBLE COUNT}

Using the Addition Law of Probability

Mutually Exclusive Events: P(A OR B) = P(A) + P(B).

Events that are NOT Mutually Exclusive: P(A OR B) = P(A) + P(B) - P(A AND B).

4 examples of calculating the probability for an OR Logical Test:

1) From Data Set
2) From Frequency Distribution
3) From Cross Tabulated Report
4) From Pre-determined Probabilities

	SN	PPM
Visitor	Space Needle	Pike's Place Market
1	Yes	Yes
2	No	Yes
3	No	Yes
4	No	No
5	Yes	Yes
6	Yes	Yes
7	No	No
8	Yes	No
9	No	Yes
10	Yes	Yes
Total Records		10

1) From Data Set for visitors to Seattle

For a randomly selected Seattle visitor calculate the probability that they visited the SN or PPM.

Condition	Yes	from Space Needle field
Count	5	=COUNTIFS(SN,D571)
Condition	Yes	from Pike's Place field
Count	7	=COUNTIFS(PPM,D573)
Condition	Yes	Visited both sites (AND)
Count	4	=COUNTIFS(SN,D575,PPM,D575)
P(SN OR PPM)	0.8	(5+7-4)/10
P(SN OR PPM)	0.8	=ROWS(FILTER(I564:J573,(SN=D575)+(PPM=D575)))/J574

2) From Frequency Distribution with Mutually Exclusive Categories

For a randomly selected Delta Airline Passenger calculate the following probabilities:

P(Early OR On Time)	0.894	(100+794)/1000
P(Cancelled)	0.025	25/1000
P(Not Cancelled)	0.975	1-0.025

Arrival	Frequency
Early	100
On Time	794
Late	81
Canceled	25
Grand Total	**1000**

3) From Cross Tabulated Report created from the visitors to Seattle data set

For a randomly selected Seattle visitor calculate the probability that they visited the SN or PPM.

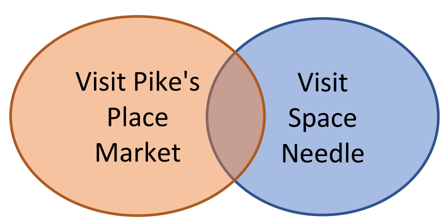
P(SN OR PPM)	0.8
	(5+7-4)/10

Frequency	Pike's Place M.		
Space N.	No	Yes	**Totals**
No	2	3	**5**
Yes	1	4	**5**
Totals	**3**	**7**	**10**

4) From Pre-determined Probabilities

For a randomly selected Seattle visitor calculate the probability that they visited the SN or PPM.

P(SN)	0.5		
P(PPM)	0.7		
P(SS AND PPM)	0.4		
P(SN OR PPM)		0.8	0.5+0.7-0.4



OR might be shown as: P(At least 1 site), or, P(1 or more sites)

Conditional Probability

The probability of an event given that another related event has already occurred.

After the first event has occurred the sample space has changed (gotten smaller).

The two events that make up the Conditional Probability are considered **dependent**.

Notation: P(A | B) = "Probability of event A, given that event B has already occurred". "|" = " given that".

<u>Examples:</u>

1) This is not conditional probability:

What is probability of pulling one Queen card from a randomly shuffled deck of 52 cards?

There are 4 matching sample points: Q♥, Q♣, Q♠, Q♦

When you calculated the probability, the full Sample Space is intact. Sample Space = 52 cards.

P(Pull 1 Queen from deck of cards) = $P(Q_1)$ = 4/52

2) This IS conditional probability:

Given that you pulled a Spade Queen card (Q♠) as your first card, what is the probability that you can pull a Queen card in the second try?

There are 3 matching sample points: Q♥, Q♣, Q♦

When you calculated the probability, the Sample Space has changed. Sample Space = 51 cards.

P(Pull second Queen given that you already pulled a Queen) = $P(Q_2 \mid Q_1)$ = 3/51

The events "Pulling a Second Queen" and "Pulling a First Queen" are dependent events.

3) Calculate the probability that a randomly selected American uses Facebook given that they use YouTube.

A random survey of American social media use was conducted and the results are presented in a cross tab report. From the report, calculate the probability that a randomly selected American uses Facebook given that they use YouTube. Said a different way: If a randomly selected American uses YouTube, what is probability that they also use Facebook?

Facebook = FB

YouTube = YT

Frequency	YouTube		
Facebook	Not Use YT	Use YT	**Total**
Not Use FB	16	46	**62**
Use FB	22	116	**138**
Total	**38**	**162**	**200**

For this problem, the frequencies are given to you, but you can not use the full Sample Space of 200. Because the question askes you to isolate your calculation to just the sample space for YouTube, the Sample Space changes, the denominator that you use is the total number of people who use YouTube, 162.

P(Use YT) =	0.81	162/200	Sample Space not change
P(Use FB AND Use YT) =	0.58	116/200	Sample Space not change
P(Use FB | Use YT) =	0.71604938	116/162	🡐 Sample Space DOES change
P(Use YT | Use FB) =	0.84057971	116/138	🡐 Sample Space DOES change

Conditional Probability Rule:

P(A | B) = P(A AND B)/P(B) = "Probability Event A occurs given that Event B has already occurred".

P(B | A) = P(A AND B)/P(A) = "Probability Event B occurs given that Event A has already occurred".

Textbook notation:

$P(A \mid B) = P(A \cap B)/P(B)$ =

$P(B \mid A) = P(A \cap B)/P(A)$ =

P(Use FB | Use YT) =	0.71604938
	0.58/0.81
P(Use YT | Use FB) =	0.84057971
	0.58/0.69

Joint Probability Table			
Probability	YouTube		
Facebook	Not Use YT	Use YT	**Total**
Not Use FB	0.08	0.23	**0.31**
Use FB	0.11	0.58	**0.69**
Total	**0.19**	**0.81**	**1**

AND %

Marginal %

Joint Probability Tables

Joint Probability Tables are cross tabulated tables that that have row and column header conditions and show AND Logical Test Probabilities (Joint Probabilities) on this inside of the table, Single Condition Probabilities (Marginal Probabilities) in the row and column total sections.

Joint Probability Table			
Probability	YouTube		
Facebook	Not Use YT	Use YT	**Total**
Not Use FB	0.08	0.23	**0.31**
Use FB	0.11	0.58	**0.69**
Total	**0.19**	**0.81**	**1**

AND % (Joint Probability)

Marginal % (Single Condition Prob.)

Examples:

1) Create Joint Probability from a proper data set

When you have a proper data set with records of data, you can use the PivotTable feature to create a Cross Tabulated Frequency Distribution with the Show Values As % of Grand Total calculation.

Survey Data:

Facebook	YouTube
Use FB	Use YT
Not Use FB	Use YT
Not Use FB	Not Use YT
Use FB	Not Use YT
Use FB	Use YT
Use FB	Use YT

194 hidden rows

Joint Probability created with PivotTable:

Joint Prob.	**YT**		
FB	**Not Use YT**	**Use YT**	**Grand Total**
Not Use FB	0.08	0.23	0.31
Use FB	0.11	0.58	0.69
Grand Total	**0.19**	**0.81**	**1**

2) If you are given a cross tabulated report, the fastest way to create a Joint Probability Table is to use a Spilled Array Formula.

Report given to you:

Facebook = FB
YouTube = YT

Frequency	YouTube		
Facebook	Not Use YT	Use YT	**Total**
Not Use FB	16	46	**62**
Use FB	22	116	**138**
Total	**38**	**162**	**200**

Steps to create Joint Probability Table:

1) Copy first report
2) Delete numbers
3) Create spilled array formula

Frequency	YouTube		
Facebook	Not Use YT	Use YT	**Total**
Not Use FB	0.08	0.23	**0.31**
Use FB	0.11	0.58	**0.69**
Total	**0.19**	**0.81**	**1**

Joint Probability Table Formula: =G875:I877/I877

Independent Events

Two events are independent if the probability of one event is not affected by the occurrence of the other.

Examples of Independent Events:

1) Rolling one die does not affect the roll of the next die.

2) Whether or not Alphabet stock (Google) goes up in a day does not affect whether or not Safeway stock goes up in that same day.

Rule of Independence

P(A | B) = P(A) , B has no affect on A.
P(B | A) = P(B) , A has no affect on B.
Otherwise the events are dependent.

Examples of the Rule of Independent:

1) P(Roll 6 on Second Roll of Die) = P(Roll 6 on Second Roll of Die | Rolled 6 on First Roll of Die) = 1/6

2) The probability of making a sale for any one sales call is 0.15. Each sales call is an independent event.
P(Sale on Call 2) = P(Sales on Call 2 | Sale on Call 1) = 0.15.

Multiplication Law of Probability (AND Logical Test / *)

Multiplication Law is used to calculate the probability of the intersection of events (probability of an AND Logical Test).

Multiplication rule for dependent events:	*Textbook notation:*
P(A AND B) = P(B)*P(A | B)	P(A ∩ B) = P(B)*P(A | B)
P(A AND B) = P(A)*P(B | A)	P(A ∩ B) = P(A)*P(B | A)
Multiplication rule for independent events:	
P(A AND B) = P(A)*P(B)	P(A ∩ B) = P(A AND B) = P(A)*P(B)

Example of Multiplying Dependent Events to calculate P(A AND B):

1) Calculate the probability that you can pull two straight Queens from a deck of cards (without replacement).

Population size (# cards in deck) = number_pop =	52
Success in Population (# Queens) = population_s =	4
Sample Size (cards pulled in successions) = number_sample =	2
Success in Sample (# Queens) = sample_s =	2

P(Pull Two Straight Queens from Deck of Cards Without Replacement) =
$P(Q_2 \text{ AND } Q_1) = P(Q_1)* P(Q_2 \mid Q_1)$ = 0.00452489 4/52*(4-1)/(52-1)

0.00452489
HYPGEOM.DIST(2,2,4,52,0)

Examples of Multiplying Independent Events to calculate P(A AND B):

If the probability for a sale for any particular sales call is 0.15, and one sales call does not affect the next, then the probability of make a sale for the first call and a sale for the second call is:
P(s)*P(s) = 0.15*0.15 = 0.0225

Rule of Independence using Multiplication:

P(A AND B) = P(A)*P(B), where events A and B are independent. Otherwise the events are dependent.

Example:

If P(G) = 0.65, P(S) = 0.35, P(G AND S) = 0.2275, are the events independent? Yes because 0.65*0.35 = 0.2275.

Mutually Exclusive vs. Independence

Don't confuse "Mutually Exclusive" (events have no sample points in common; if one event occurs, the other did not) and "Independence" (the two events exist, but are not related). Two non-zero probabilities cannot be both mutually exclusive and independent: Independence means two events exist, but are not related; whereas, Mutual Exclusivity means when one event occurs, the other cannot.

<u>Using the Multiplication Law of Probability</u>

Multiplication rule for dependent events:

P(A AND B) = P(B)*P(A | B)

P(A AND B) = P(A)*P(B | A)

Multiplication rule for independent events:

P(A AND B) = P(A)*P(B)

<u>4 examples of calculating the probability for an AND Logical Test:</u>

1) Calculate the probability that both Alphabet stock and Safeway stock will go up next year.
2) From a Cross Tabulated Report on American Social Media create a Probability Tree on worksheet.
3) From a Cross Tabulated Report on American Social Media create a Probability Tree on paper.
4) From a Cross Tabulated Report on Heart Attack & Smoking create a Probability Tree on paper.

1) The probability that Alphabet stock (Google) will go up next year is 0.65 and the probability that Safeway stock will go up next year is 0.35. What is probability that both will go up. Assume events are independent.

P(Alphabet stock will go up in 2023)	0.65	
P(Safeway will go up in 2023)	0.35	
P(Both will go up in 2023)	0.2275	0.65*0.35

2) A random survey of American social media use was conducted. The random experiment will be to first ask: "Do you use YouTube, Yes, or No?". The second question will be to ask: "Do you use Facebook, Yes, or No?". From a Cross Tab Report shown below, create a Probability Tree that shows Root (Do you use YouTube?), Conditional (Do you use Facebook?) and Joint Probabilities (Do you use both?).

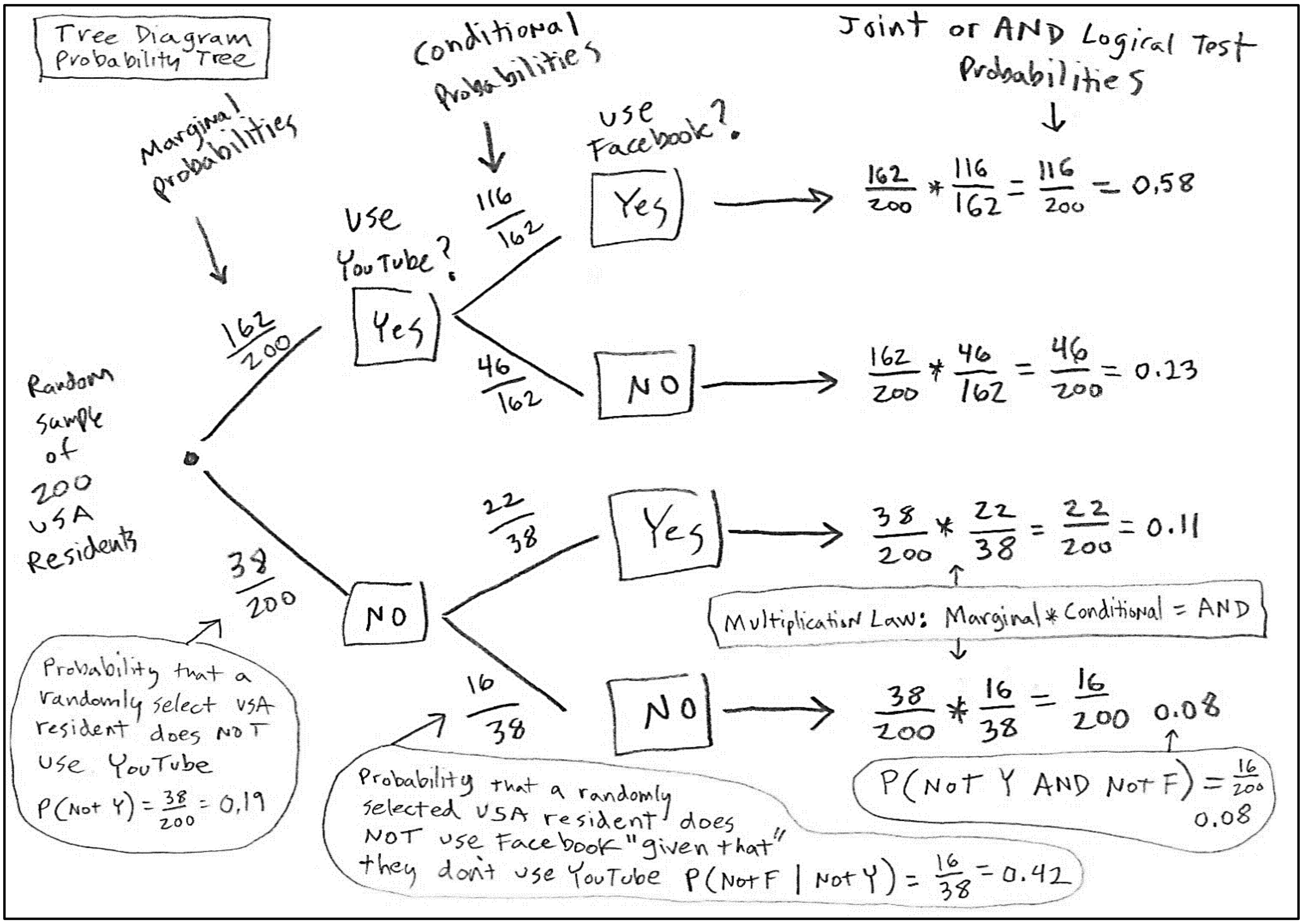
Frequency	YouTube		
Facebook	Not Use YT	Use YT	**Total**
Not Use FB	16	46	**62**
Use FB	22	116	**138**
Total	**38**	**162**	**200**

Multiplication Law of Probability

Marginal Probabilities			**Conditional Probabilities**			**Joint Probabilities (AND)**	
Use YouTube?			**Use Facebook?**			**Use Both**	
			Yes =	116/162	0.71604938	162/200*116/162 = 116/200	0.58
Yes =	162/200	0.81					
			No =	46/162	0.28395062	162/200*46/162 = 46/200	0.23
			Yes =	22/38	0.57894737	38/200*22/38 = 22/200	0.11
No =	38/200	0.19					
			No =	16/38	0.42105263	38/200*16/38 = 16/200	0.08

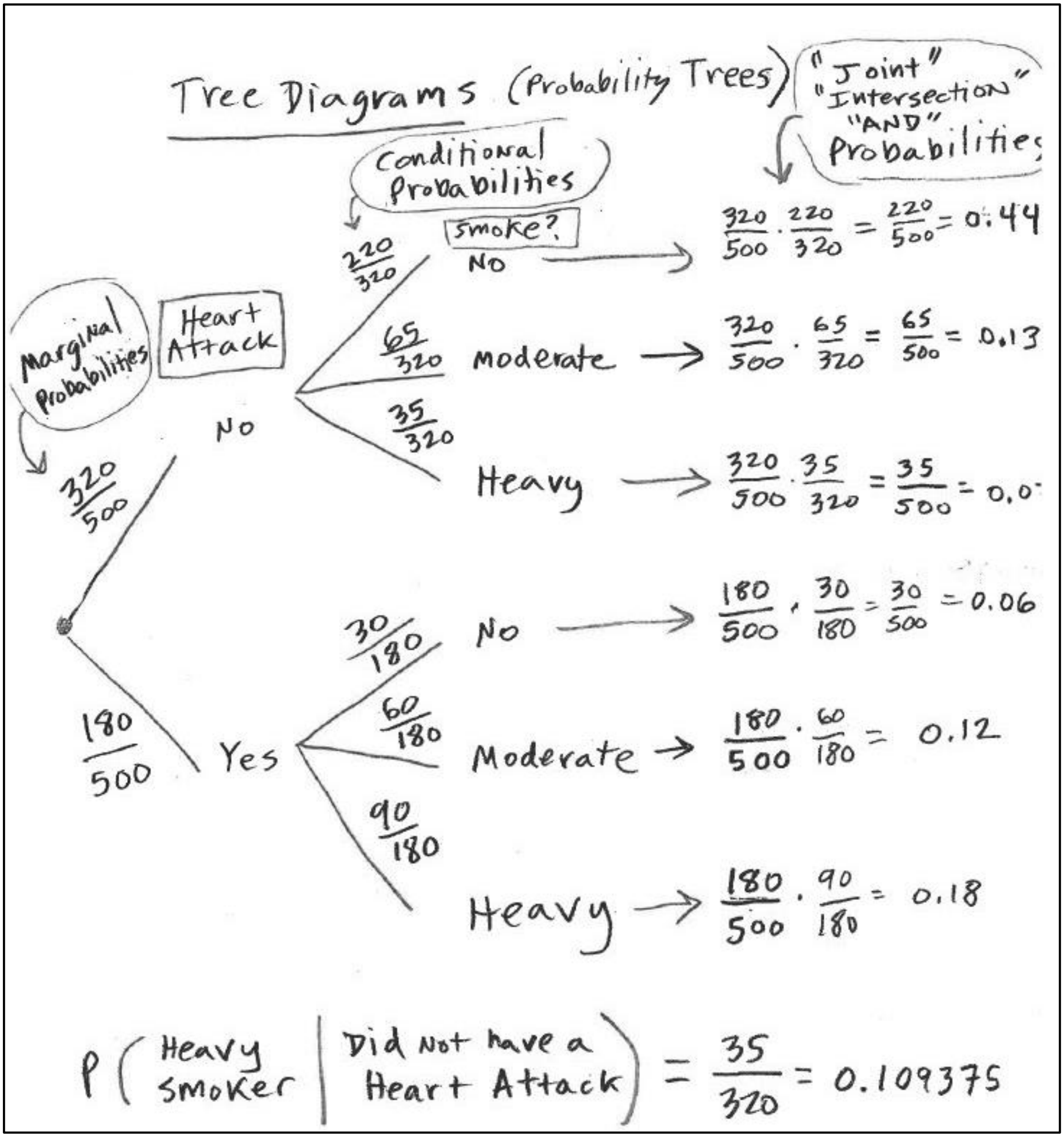
3) A random survey of American social media use was conducted. From a Cross Tab Report, create a Probability Tree that shows Root, Conditional and Joint Probabilities on paper:

Frequency	YouTube		
Facebook	Not Use YT	Use YT	**Total**
Not Use FB	16	46	**62**
Use FB	22	116	**138**
Total	**38**	**162**	**200**



4) A random survey of Americans concerning heart attacks and smoking was conducted. From a Cross Tabulated Report on Heart Attack & Smoking create a Probability Tree on paper.

Frequency	Heart Attack		
Smoke	Yes	No	**Total**
No	30	220	**250**
Moderate	60	65	**125**
Heavy	90	35	**125**
Total	**180**	**320**	**500**



Bayes' Theorem

Used when we have initial probabilities, called Prior Probabilities, we are given or get new information, and we want to revise or update the prior probabilities by calculating Posterior Probabilities.

Prior probabilities (subjective) → New information (Relative Frequency) → Apply Bayes' Theorem → Posterior (Revised) probabilities

Prior probabilities: you estimate the probability that you can get a passing CPA score of 750 or more points as

$P(\geq 750) = 0.25$

New information: New info = survey Report that asked people who took CPA:

1. what was their score
2. Did they take preparation course

Report said:

$P(PC \mid \geq 750) = 0.60$

$P(PC \mid < 750) = 0.31$

Apply Bayes' Theorem:

$$\frac{P(\geq 750 \text{ AND } PC)}{\left(P(\geq 750 \text{ AND } PC) + P(<750 \text{ AND } PC)\right)}$$

$$\frac{0.15}{0.15 + 0.2325}$$

Posterior probabilities:

$P(\geq 750 \mid PC) = 0.39$

Revised → Posterior Probabilities → Revised

Bayes' Theorem Formula (2 Event case)

Prior Event 1 = A_1

Prior Event 2 = A_2

Event = B

$$P(A_1 | B) = \frac{P(A_1 \text{ AND } B)}{P(A_1 \text{ AND } B) + P(A_2 \text{ AND } B)}$$

$$P(A_2 | B) = \frac{P(A_2 \text{ AND } B)}{P(A_2 \text{ AND } B) + P(A_1 \text{ AND } B)}$$

or

$$P(A_1 | B) = \frac{P(A_1) * P(B | A_1)}{P(A_1) * P(B | A_1) + P(A_2) * P(B | A_2)}$$

$$P(A_2 | B) = \frac{P(A_2) * P(B | A_2)}{P(A_2) * P(B | A_2) + P(A_1) * P(B | A_1)}$$

Using Multiplication Law and Bayes' Theorem, Tabular Method can be used:

Example 1:

You're about to take CPA exam, and you heard from last exam that the pass rate was 25%

P(Pass)		0.25	
P(Not Pass)		0.75	1 - 0.25

Your plan is to take a preparation course before taking the CPA exam, and therefore you would like to know the probability P(Pass | Took Prep Course).

Then you get new information from a survey that asked people who passed exam, whether or not they took a Preparation Course for the CPA exam:

P(Took Prep Course | Pass)	0.60
P(Took Prep Course | Not Pass)	0.31

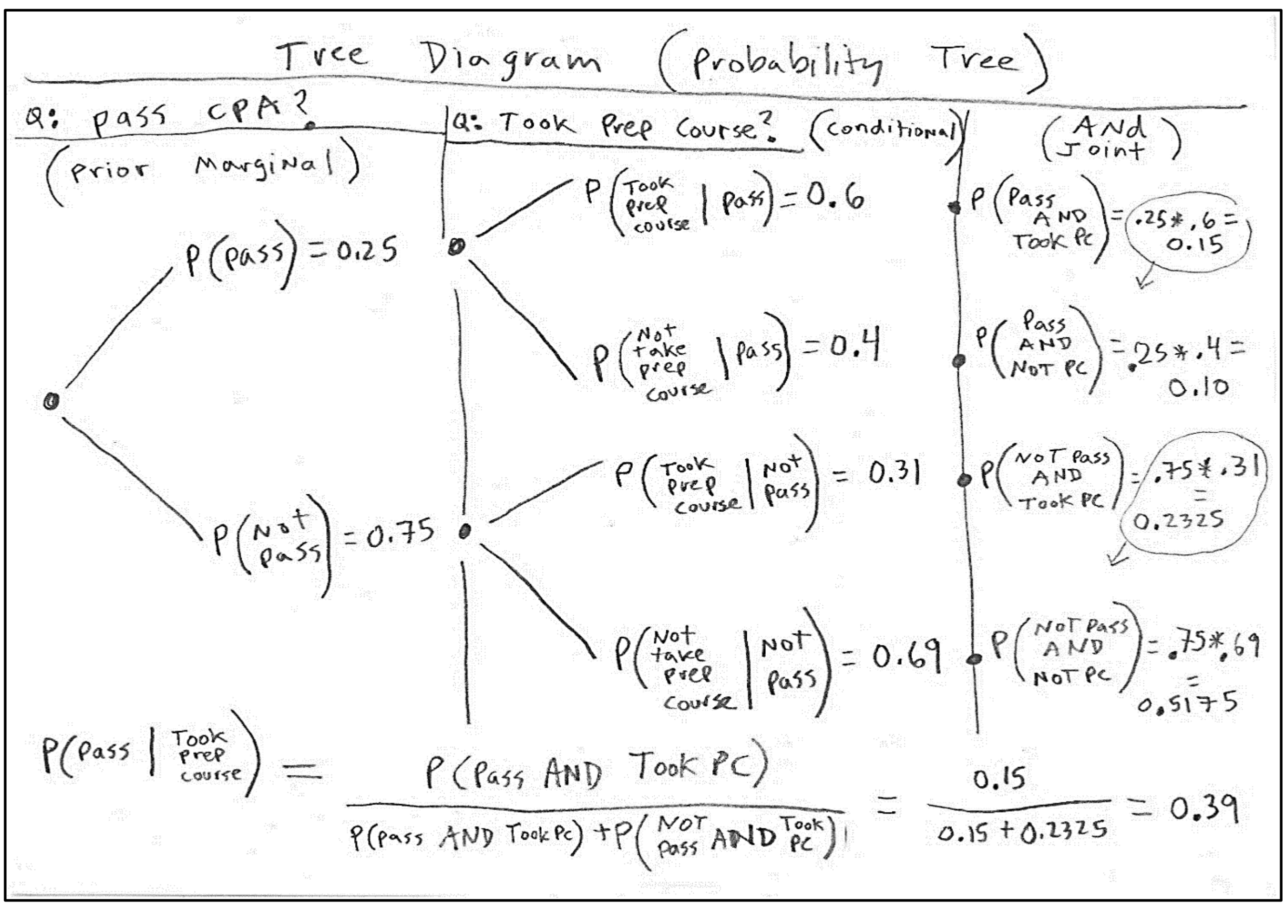
For this textbook, these probabilities are given				Use Multiplication Rule to get AND		
Prior (Marginal)		**Conditional**		**Joint (AND)**		
P(Pass)	0.25	P(Took Prep Course | Pass)	0.6	P(Pass AND Took Prep Course)	0.15	0.25*0.6
P(Not Pass)	0.75	P(Took Prep Course | Not Pass)	0.31	P(Not Pass AND Took Prep Course)	0.2325	0.75*0.31
Total	1			Total = Probability Took Prep Course =>	0.3825	

Use "Concept: compare parts to whole", or Conditional Probability Rule.

Posterior (Revised Probabilities)

P(Pass | Took Prep Course)	0.39215686	0.15/0.2325
P(Pass | Took Prep Course)	0.39215686	

0.25*0.6/SUMPRODUCT({0.25;0.75},{0.6;0.31})



If we had full data set, then we can easily use the PivotTable tool to create a Joint Probability Table and then create the conditional probability needed for Bayes Theorem:

	A	B	C	D	E	F	G	H
1								
2		**Survey of people who took CPA exam:**						
3		**Q1:** What was your score (0-1000)?						
4		**Q2:** Did you take a Preparation Course (PC or NPC)?						
5								
6		Score	Prep Course		P(Not Pass | Took PC) =		0.60784	0.2325/0.3825
7		927	Not Take Prep Course		P(Pass | Took PC) =		0.39216	0.15/0.3825
8		343	Not Take Prep Course					
9		757	Took Prep Course					
10		834	Took Prep Course					
11		641	Not Take Prep Course		**Joint Probability**	**Prep Course**		
12		204	Not Take Prep Course		**Pass?**	**Not Take PC**	**Took PC**	**Grand Total**
13		139	Not Take Prep Course		Not Pass	51.75%	23.25%	75.00%
14		475	Not Take Prep Course		Pass	10.00%	15.00%	25.00%
15		197	Took Prep Course		**Grand Total**	**61.75%**	**38.25%**	**100.00%**
16		757	Took Prep Course					
17		143	Not Take Prep Course					
18		46	Not Take Prep Course					
19		108	Not Take Prep Course					
20		544	Took Prep Course					
21		535	Not Take Prep Course					
22		363	Not Take Prep Course					
23		891	Took Prep Course					
24		891	Took Prep Course					
25		145	Not Take Prep Course					
26		688	Not Take Prep Course					
27		420	Not Take Prep Course					
28		817	Not Take Prep Course					
9997		728	Took Prep Course					
9998		671	Not Take Prep Course					
9999		574	Took Prep Course					
10000		463	Not Take Prep Course					
10001		372	Not Take Prep Course					
10002		581	Not Take Prep Course					
10003		949	Not Take Prep Course					
10004		367	Not Take Prep Course					
10005		470	Not Take Prep Course					
10006		374	Took Prep Course					

Example 2:

Bayes' Theorem Example 2

prior probability } Event = Leave Garage Door Open ⇒ $P(GO) = 0.20$

conditional probabilities } Event stuff stolen given Garage open ⇒ $P(SS|GO) = 0.05$

Event Stuff stolen given Garage NOT open ⇒ $P(SS|GNO) = 0.01$

we want to know what is probability that Garage was open given stuff stolen $P(GO|SS)$

Prior	conditional	Joint AND	Posterior
$P(GO)$ 0.20	$P(SS\|GO)$ 0.05	$P(GO \text{ AND } SS)$ $0.2 * 0.05$ $= 0.01$	$P(GO\|SS) = \frac{0.01}{0.018}$ $= 0.556$
$P(GNO)$ $1 - 0.20$ $= 0.80$	$P(SS\|GNO)$ 0.01	$P(GNO \text{ AND } SS)$ $0.8 * 0.01$ $= 0.008$	$P(GNO\|SS) = \frac{0.008}{0.018}$ $= 0.444$
		$\Sigma = 0.01 + 0.008 = 0.018$	↑ parts compared to whole

whole → 0.018

$P(SS) = 0.018$

therefor

$P(\text{Not } SS) = 1 - 0.018 = 0.982$